Essays in Computation of Heterogeneous Agent Models

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Dedication

To my dearest wife, without whose support none of this would have been possible. To my parents and brother, who inspired me to reach higher. To Vasudevan, Madhavan, Shridharan, and Shridevi, who make it all worthwhile.

Abstract

This dissertation consists of three chapters. The first chapter examines Robert Shimer's 2005 paper and the important puzzle in labor economics it documented: Conventional models which use Mortensen-Pissarides undirected search with Nash Bargaining over wages are unable to match the volatility of market tightness seen in the data. Although some headway has been made in resolving the puzzle, the main solutions have involved fundamental changes to the base framework of Shimer's original model via changes in both bargaining and search protocols. This paper resolves the puzzle within the original framework through the introduction of agent heterogeneity.

The second chapter presents a model of international transmission of financial shocks where the country of origin is fundamental to the transmission of the shock. A country is defined by the quality of its financial markets, with financially-developed countries better able to insure against idiosyncratic shocks. Highly developed countries tend to accumulate larger positions in riskier, but more productive, capital flows, as seen in the data. When a financial shock occurs, the ability to insure is impaired, which lessens demand for risky foreign capital, which lowers production abroad. We interpret the Financial Crisis of 2008 as a change in the ability of financial market quality and calibrate the model to match the change in capital flows. Importantly, the calibrated model matches not only changes in capital flows, but also relative movements in interest rates as well as changes in debt flows.

The third chapter proposes a modification to the endogenous grid method that allows it to be used in multi-dimensional problems. It provides a background on interpolation problems, and various standard solutions. The paper then uses a Gaussian basis function for estimation of the standard economics utility problems, and compares the performance of this modification to standard methods of value function iteration. The solution method yields higher accuracy for lower grid point levels, but suffers from slower performance as the number of grid points increases. Further work is required to investigate the effectiveness of alternative basis functions for a variety of other utility forms.

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Chapter 1

The Shimer Puzzle under Heterogeneous Agents

1.1 Introduction

In his seminal paper, "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", Robert Shimer details a puzzle that still poses difficulties today - the ratio of job vacancies to unemployment (market tightness) is an order of magnitude higher in the data than what standard models predict, and they react immediately to a productivity shock. I develop two search and matching model with heterogeneous households.

In the first model, we use an overlapping generations model with history dependent heterogeneity. In addition, wage negotiations between firms and households are costly, and therefore only occur if there is sufficient surplus over which to bargain. These two components increase the volatility of firm-worker matches and slow down the response of market tightness, and so are able to replicate movements seen in data. The mechanism through which these factors operate is fairly intuitive. Consider, for example, the case where negotiation costs are so high that the surplus from a newly formed match is close to zero. The benefits of a positive productivity shock will go to the firm, since any renegotiation of wages would reduce surplus enough to *lower* wages for the worker and surplus for the firm. Similarly, much of the cost of a negative productivity shock will also go to the firm, as they cannot reduce wages in response, though this is bounded below by zero. As vacancy postings are directly related to levels of firm surplus, an increased volatility in surplus implies an increased variance of vacancy postings.

The second model introduces skill heterogeneity across workers, with low productivity individuals producing less than their outside option. This creates an extensive margin for the labor decision, and results in endogenous separations between workers and firms. When a shock occurs, a mass of individuals enter the labor force as unemployed individuals. While the total mass of these workers may be low, their mass relative to unemployed workers can be high. As a result, a large number of vacancies will be posted in response. Additionally, as the value to a firm of the worker at the labor force cutoff is zero, the expected value of a match is lower in this model than in the standard formulation - where the value is always positive. These two factors result in significantly stronger labor responses to productivity shocks.

The main contribution of this paper is to introduce, into a typical Mortensen and Pissarides (1993) search model (hereafter MP), tractable changes that are able to reproduce the volatility of unemployment and vacancies documented in Shimer (2005). My approach to heterogeneity is novel in the fact that, in both models, it is completely observed by both firms and households. Additionally, both models generate wage rigidity[1] in a way that is new to this literature. In the first model, this is accomplished by allowing agents to agree not to negotiate wages [2], and in the second model, this occurs due to the entry of low-productivity workers which reduces the expected surplus of a match. Finally, in comparison to earlier solutions, this model is robust to procyclical changes in home production that Chodorow-Reich and Karabarbounis (2013) note eliminate much of the volatility generated by previous solutions.

For the first model, while wage rigidity is the basis for the model's increased volatility, in a reasonably calibrated model, it alone is insufficient to match the movements seen in the data. Household types, habits, and risk aversion further amplify this mechanism, however, even when allowing firms to have perfect information regarding potential workers. To see how agent types may help, consider a two period lifecycle model with iid shocks. Wages are a function of both current productivity and expected productivity in the next period. For the old generation, a one period shock will translate directly to wages, and so, if firms only hired old workers, they would have no incentive to adjust vacancies. On the other hand, this shock has a lower effect on the young generation, as it does not affect their productivity

[1] Haefke et al. (2013) note a difference between wage stickiness and wage rigidity. From their paper, wage stickiness denotes " an explicitly modeled friction that prevents wages from adjusting to the level that would otherwise be obtained." Wage rigidity occurs when " the observed response of wages to changes in productivity in the data being smaller than one." I will keep this distinction in this paper.

[2] The mechanism I use to generate wage rigidity is a cost to negotiation. By the definition in Haefke et al. (2013), this is wage rigidity, and not wage stickiness. Nothing prevents agents from negotiating, they simply choose not to as both parties are better off by not doing so.

next period. The value to a firm of having a young worker goes up after this shock, and because of free entry, this results in firms posting more vacancies.

This paper is obviously not the first to attempt to resolve this puzzle. In fact, in its conclusion, Shimer (2005) posits several potential solutions: wage rigidity and information asymmetry. Much of the literature in the subsequent decade has focused on one of these two channels. Hagedorn and Manovskii (2008), however, called into question Shimer's result, noting that it depended heavily on the calibration of the model. Specifically two key parameters drove most of the result: the worker's negotiation power, γ, and the value of non-market activity (i.e unemployment insurance or home production), z. Calibrating their model, they find a value of z equal to 95 percent of the value of job output and a bargaining power of 0.03. Under this specification, the authors are able to closely match the data statistics documented in Shimer (2005).

The Hagedorn and Manovskii (2008) calibration generates an extremely high wage share. This has implications on the response of the model to policy changes. Hornstein et al. (2005) show that a 15 percent rise in benefits would double the unemployment rate, larger than is seen in US data. This resulted is supported by Costain and Reiter (2008), who find that the elasticity of unemployment with respect to an increase in benefits is almost seven times higher in the Hagedorn and Manovskii (2008) calibration than in estimates obtained using cross-country data.

Wage rigidity as a solution to the puzzle has been introduced in several ways. One of the earliest, Hall (2005), changes the bargaining process from Nash Bargaining to a Nash Auction. Hall and Milgrom (2008) use Rubinstein bargaining to obtain the required wage stickiness. In fact, Hall and Milgrom (2008) is spiritually close to the solution I introduce. Rather than having a delay cost to the employer for bargaining, however, I introduce a loss to surplus that affects both firms and workers. If both parties agree not to negotiate, this loss can be avoided - otherwise, it must be incurred.

Chodorow-Reich and Karabarbounis (2013) identifies a new issue with both Hagedorn and Manovskii (2008) and Hall and Milgrom (2008). While both papers are able to match the market tightness volatility in response to a productivity shock, they fail to do so when presented with a surplus shock. That is, if the workers outside option is highly correlated with their productivity, a shock to productivity also increases the workers outside option of not working. Under this scenario, neither paper can match the data. Chodorow-Reich and Karabarbounis (2013) go on to show that productivity and z are highly correlated, and call into question the results of these papers.

Gertler and Trigari (2009) explicitly introduce wage rigidity in the form of multi-period wage contracts. New hires are subject to the wages of existing workers of similar productivity and firms face a quadratic adjustment cost when changing employment. By setting the frequency of negotiations to be on average once per three quarters, the paper is able to quantitatively match the data. My model can be seen as a generalization of this process, where the decision to negotiate is endogenous rather than exogenous.

While wage stickiness seems like a positive avenue through which to explore the puzzle, it is not without its issues. Haefke et al. (2013) uses worker-level data to find that aggregate wages are quite rigid while the wages of new hires are not. However, they note that only a small amount of rigidity is required to match the response of job creation to productivity changes. As such, while their results eliminate solutions that require a large amount of wage stickiness to generate rigidity, they are consistent with the rigidity techniques used by Hall and Milgrom (2008) or Hagedorn and Manovskii (2008).

The second main branch of investigation into the puzzle involves unobserved heterogeneity. Robin (2010), building on Postel-Vinay and Robin (2002), introduces unobserved heterogeneity in worker ability. Wages are also set by long term contracts, and only renegotiated by mutual consent. Negative shocks cause workers with low productivity to become unemployed which amplifies the response of unemployment and vacancies to productivity shocks.

Menzio and Shi (2011) considers a model of directed on-the-job search where there is heterogeneity in the quality of firm-worker matches. They show that productivity shocks generate large fluctuations in transitions, unemployment and vacancies only when match quality is observed *after* the match occurs.

Lise and Robin (2013) combine both these literatures, building a model with heterogeneous firms and workers, on the job search, and a new wage mechanism. By showing that the worker-firm surplus is independent of the aggregate state, they show their model can match the employment and unemployment movements seen in the data. I obtain similar results without altering the wage neogtiation process and only having worker heterogeneity.

The structure of the paper is as follows. In 1.2, I present a model with overlapping generations. Further, I provide its solution and conduct a comparative statics experiment to test the response of market tightness to both productivity and surplus shocks. In 1.3, I present a model with an endogenous labor choice along with the results of several comparative statics tests. 1.4 provides a brief conclusion.

1.2 Model with Overlapping Generations

1.2.1 Environment

I consider a discrete time model with lifecycles. Each period, the oldest generation is replaced by a young generation of the same size, implying no population growth. Furthermore, the youngest generation has the same distribution across employment states as the aggregate economy. Individuals form habits based on employment history. Finally, workers freely gain skills as they age, and so older generations are naturally more productive than younger ones.

As is traditional in MP models, I have a continuum of firms, each with a single job. Firms are identical, and production is determined by some exogenous technology that is subject to aggregate productivity shocks, z. As a result, output depends solely on the productivity of the worker and the aggregate state. As there is no commitment, wages are renegotiated every period at a cost, unless both parties agree not to do so. There is no on-the-job search, and matching is undirected (i.e. firms cannot look for specific types).

The timeline of the model is as follows:

1. Aggregate productivity shock, z, is realized
2. Exogenous separations occur, with probability s.
3. Firms post vacancies
4. Matching and Production take place simultaneously

Matching

All matches between firms and workers occur in a matching market. There is a continuum of workers in the economy with unit mass, and similarly, a continuum of potential firms with infinite mass.[3] In period t, u_t represents the mass of individuals searching for a job, and v_t represents the mass of firms posting a position. Workers who lose their job in period t can search for a position in the same period. Workers matched in period t start producing in period t+1. Posting a position costs the firm c, and the posting is valid for a single period.

[3] This allows for firm entry and exit into the market.

The flow of matches within a period is given by the following matching function

$$m\left(u_t, v_t\right) = \frac{\mu u_t v_t}{\left(u_t^\eta + \left(\mu v_t\right)^\eta\right)^{1/\eta}} \tag{1.1}$$

This equation is closely related to den Haan et al. (1997), but includes the extra parameter, μ, that enables targeting of match elasticity.

Market tightness, θ, is defined as the ratio of unemployment to vacancies (v/u). The matching function is homogeneous of degree one, and can be restated in terms of market tightness.

$$u_t m\left(1, \frac{v_t}{u_t}\right) = u_t m\left(1, \theta_t\right) = u_t \frac{\mu\theta_t}{\left(1 + \left(\mu\theta_t\right)^\eta\right)^{1/\eta}}$$

Vacant jobs and unemployed workers are randomly matched each period. If multiple applicants apply to the same position, the firm chooses one of the applicants randomly, where each worker has the same probability of being selected. In this manner, though firms will observe differences in individual applicants, they will not select them according to their type. The probability of an unemployed worker becoming employed, $f(\theta)$, is given by:

$$f(\theta) \equiv \frac{u_t m\left(1, \theta_t\right)}{u_t} = m\left(1, \theta_t\right) \tag{1.2}$$

and the probability of a firm filling a position, $q(\theta)$, is

$$q(\theta) \equiv \frac{u_t m\left(1, \theta_t\right)}{v_t} = \frac{f\left(\theta_t\right)}{\theta} \tag{1.3}$$

Unemployment Transition

Given the timing of decisions, the number of individuals who are unemployed is not equal to the number of people who are job hunting. Specifically, if u is the unemployment rate entering the period, then the number of households searching for a job is given by the sum of unemployed workers entering the period and those workers who were exogenously separated from their position

$$u_h = u + s(1-u)$$

The new unemployment rate entering the next period is given by:

$$u' = \left(1 - f(\theta)\right) u_h$$

Note that the denominator for market tightness in this model is u_h and not u.

Worker Problem

Workers exist in one of two states, employed or unemployed. Unemployed workers have a home production output of b(z), whereas employed workers earn a wage w(z), both of which *may* be functions of the aggregate shock. The elasticity of b with respect to z will be a model parameter.

Households follow a lifecycle process and are indexed by age, denoted by $i \in I$. Each worker has developed a habit, x, based on their work history. I consider habit development as rungs on a ladder, where being employed increases your habit by one step and being unemployed decreases it by the same. All individuals of the youngest generation start at the same habit level. Therefore, the evolution of habit is given by:

$$\begin{aligned} \text{Unemployed: } x'_u &= x - 1 \\ \text{Employed: } x'_e &= x + 1 \end{aligned} \tag{1.4}$$

To allow for non-linear utilities, the support of x is $[0 \ldots b(z_{min})]$.

The value of being unemployed in state z, to a worker of generation i, is given by

$$\begin{aligned} U_i\left(z,x\right) = u_i\left(b(z),x\right) + \beta\mathbb{E}_z\Big[&U_{i+1}\left(z',x-1\right) + \\ &f\left(\theta_z\right)\left\{E_{i+1}\left(z',x-1,b(z')\right) - U_{i+1}\left(z',x-1\right)\right\}\Big] \end{aligned} \tag{1.5}$$

u_i is a quasi-concave function that is increasing in its parameters and decreasing in habit. $E_i(z,x,\bar{w})$ is the value of being employed, to a worker of generation i, with habit x, and wage in the previous period of $\bar{w}$. This value is given by

$$\begin{aligned} E_i\left(z,x,\bar{w}\right) =& u_j\left(w_j(z,\bar{w}),x\right) + \\ &\beta\mathbb{E}_z\left[E_{i+1}\left(z',x+1,w_j\right) - s\left\{E_{i+1}\left(z',x+1,w_j\right) - U_{i+1}\left(z',x+1\right)\right\}\right] \end{aligned} \tag{1.6}$$

Note that the negotiated wage for the current period, w_i, is a function of the previous period's wage.

Firm Problem

All firms have a single position and access to the same exogenous production technology. The productivity, $y_i(z)$, of a match is dependent on worker type, i, and the aggregate shock, z. Firms incur a cost when wages are negotiated, including when they are first matched with a worker. Firms are assumed to be risk neutral, so the value to a firm matched with an agent of generation i with habit x is

$$J_i(z,x,\bar{w}) = y_i(z) - w_i(z,x,\bar{w}) - \mathbb{I}_{(w_i,\bar{w})}a + \beta\mathbb{E}_z\left[sV(z') + (1-s)J_{i+1}(z',x+1,w_i)\right] \tag{1.7}$$

When the position in the firm is vacant, the value to the firm is given by

$$V(z) = -c + \beta\mathbb{E}_z\left\{q(\theta_z)\mathbb{E}J(z',x,b(z')) + (1-q(\theta_z))V(z')\right\} \tag{1.8}$$

where

$$\mathbb{E}J(z',x,b) = \sum_i \frac{1}{\#J}\int_{x_i}\int_{z'} J_i(z',x,b)\,dz'dx_i$$

Shocks affect the distribution of habits. Under the most general case, this distribution is not stationary, leading to difficulties in obtaining a numerical solution. Instead, I will assume shocks are unexpected each period, but once realized, the effect on the distribution of habits is completely known.

Free entry of firms results in

$$V(z) = 0, \forall z \tag{1.9}$$

Shock Process

The aggregate shock process is the same as described in Shimer (2005) with the appropriate extensions to account for agent type. More specifically, a shock is a percentage change in productivity, and therefore hits all agent types in the same way.

Consider a Poisson process with arrival rate λ, and let $x \in X$ be a random variable that is subject to shocks from this process. For any grid size, n, we have

$$x \in X \equiv \{-n\Delta, -(n-1)\Delta, \ldots, 0, \ldots, (n-1)\Delta, n\Delta\}$$

where $\Delta > 0$ is how far a single shock moves the random variable. The direction of movement from a shock is given by

$$z' = \begin{cases} z+\Delta, & \text{w/ prob} = \frac{1}{2}\left(1-\frac{z}{n\Delta}\right) \\ z-\Delta, & \text{w/ prob} = \frac{1}{2}\left(1+\frac{z}{n\Delta}\right) \end{cases}$$

Shimer (2005) shows that this converges to an Ornstein-Uhlenbeck process, and has the desirable property that one can move from a coarse grid to a fine grid without significantly changing estimates.

Output for type i in this model is given by

$$y_i(z) = b^* + e^z(y_i^* - b^*) \tag{1.10}$$

where y_i^* is the measure of long run, average, productivity for agents of generation i and b^* is the long run, average, outside option. Note that this is cross-sectional productivity of a specific age, and not that of an individual.

If we define

$$\zeta(z) = \frac{\mathbb{E}y(z)}{\mathbb{E}y} \tag{1.11}$$

then the outside option, b(z) is given by

$$b(z) = \zeta(z)b^* \tag{1.12}$$

Separation

The last variable of interest is the exogenous separation rate, s, which will be constant across aggregate shocks.

Wage Determination

Wage is determined by Nash bargaining, and as firms do not have commitment, wages are negotiated each period. I depart from the standard literature by adding a fixed cost that is incurred whenever bargaining occurs. Bargaining can be unilaterally forced by either firms or workers, but if the cost of negotiation is too high, and if both sides agree, they can forego negotiations and keep wages from the previous period.

Firms and workers have perfect information regarding worker type, habit, and previous period wages. When either the worker or the firm chooses to renegotiate wages, the new wage is the solution to the Nash Bargaining problem

$$\max_w \big(E_i(z,x,b) - U_i(z,x)\big)^\gamma \big(J_i(z,x,b)\big)^{1-\gamma} \tag{1.13}$$

$$st$$

$$S_i(z,x,b) = E_i(z,x,b) - U_i(z,x) + J_i(z,x,b)$$

where γ is the bargaining power. If the household has linear utility in wages, then the solution is given by

$$E_i - U_i = \gamma S_i \quad \text{and} \quad J_i = (1-\gamma) S_i \tag{1.13a}$$

If the household as non-linear utility in wages, then the solution is given by

$$\begin{aligned} E_i - U_i =& \frac{\gamma \frac{\partial E_i}{\partial w_i}}{1-\gamma+\gamma\frac{\partial E_i}{\partial w_i}} S_i, \quad \text{and} \\ J_i =& \frac{1-\gamma}{1-\gamma+\gamma\frac{\partial E_i}{\partial w_i}} S_i \end{aligned} \tag{1.13b}$$

1.2.2 Equilibrium

Characterization. Let $\mathbf{z}$ be the history of shocks and s be the exogenous separation rate. Then, given s and $\theta(\mathbf{z})$, an equilibrium is a wage profile, $\mathbf{w}$, and vacancy profile, $\mathbf{v}$, such that for all $i \in I$, $z \in Z$, $w \in \mathbf{w}$, equations (1.5), (1.6), (1.7), (1.8), (1.9), (1.10), and (1.13) are satisfied.

The solution to Nash Bargaining is the key equation, and wages are set such that the appropriate solution holds with equality whenever wages change. Importantly, this equation need not hold if both the firm and the worker agree not to renegotiate wages. In this case, additional surplus may accrue to either the firm or the worker, depending on the direction of the productivity shock.

Solving with Linear Utilities

With linear utilities, equations (1.5) and (1.6) are given by

$$\begin{aligned} U_i\left(z,x\right) = b(z) - x + \beta \mathbb{E}_z \big[& U_{i+1}\left(z',x-1\right) + \\ & f\left(\theta_z\right)\left\{E_{i+1}\left(z',x-1,b(z')\right) - U_{i+1}\left(z',x-1\right)\right\}\big] \end{aligned} \tag{1.14}$$

$$\begin{aligned} E_i\left(z,x,\bar{w}\right) = & w_i(z,x,\bar{w}) - x + \\ & \beta\mathbb{E}_z\left[E_{i+1}\left(z',x+1,w_i\right) - s\left\{E_{i+1}\left(z',x+1,w_i\right) - U_{i+1}\left(z',x+1\right)\right\}\right] \end{aligned} \tag{1.15}$$

Negotiated Wages. If wages are negotiated in a period, then the expression for the bargained wage is given by:[4]

$$w_i\left(z,x\right) = \left(1-\gamma\right)b(z) + \gamma y_i\left(z\right) - a + \gamma\beta\mathbb{E}_z\left\{f\left(\theta_z\right)J_{i+1}\left(z',x-1,b(z')\right)\right\} \tag{1.16}$$

[4] See Appendix A.1.1 for details.

Firms pay to workers a portion of the benefit they obtain from the worker not quitting every period.

Actual Wages. Given the costly nature of bargaining, it is only undertaken when one of the two parties is better off by doing so. The actual wage response to a shock, then, is given by:

$$w_i(z,x,\bar{w}) = \begin{cases} w_i(z,x), & \begin{array}{l}\text{if } E_i(z,x,w_i(z,x)) > E_i(z,x,\bar{w}) \\ \text{or } J_i(z,x,w_i(z,x)) > J_i(z,x,\bar{w})\end{array} \\ \bar{w}, & \text{otherwise} \end{cases} \tag{1.17}$$

For any given household with habit x, E_i is increasing in w and J_i is decreasing in w. Given i and x, then, there exists a lower cutoff wage, $\underline{w}_i$, such that for any $\bar{w} < \underline{w}_i$, firms choose to negotiate. Similarly, there exists an upper cutoff wage, w_i^*, such that for any $\bar{w} < w_i^*$, households choose to negotiate.

Solution. Equation (1.8), along with the free entry condition (1.9), provides the equality that must hold for any aggregate shock. With wages given by equation (1.17), for any $\theta(z)$, each of the terms in (1.8) can be solved.

Solving with Non-Linear Utilities

With non-linear utilities, equations (1.5) and (1.6) are given by

$$\begin{aligned} U_i\left(z,x\right) = u(b(z)-x) + \beta\mathbb{E}_z\Big[U_{i+1}\left(z',x-1\right) + \\ f\left(\theta_z\right)\left\{E_{i+1}\left(z',x-1,b(z')\right) - U_{i+1}\left(z',x-1\right)\right\}\Big] \end{aligned} \tag{1.18}$$

$$\begin{aligned} E_i\left(z,x,\bar{w}\right) = u(w_i(z,x,\bar{w})-x) + \beta\mathbb{E}_z\Big[E_{i+1}\left(z',x+1,w_i\right) - \\ s\left\{E_{i+1}\left(z',x+1,w_i\right) - U_{i+1}\left(z',x+1\right)\right\}\Big] \end{aligned} \tag{1.19}$$

Wages. We cannot solve for wages directly as wage appears non-linearly in (1.18) and (1.19). Instead, if wages are negotiated in a period, the wage is the solution to the system of non-linear equations given by equation (1.13b), where I have one equation for each generation, habit, and potential wage entering the period. The actual wage observed is still given by (1.17).

Solution. The rest of the solution can be found using the same method as for linear utilities.

1.2.3 Comparative Statics

To see if market tightness responds sufficiently to aggregate shocks, I first consider some comparative statics. This is done by assuming that there are no expected aggregate shocks, and then evaluating the elasticity of θ with respect to a shock. To find this value, first restate equation (1.8) as

$$\frac{c}{\beta} = \frac{f(\theta_z)}{\theta_z}\mathbb{E}_z J_i\left(z', x, b(z')\right)$$

where we eliminate V due to the free entry condition.

The question, then, is with respect to what kind of shock should we evaluate the elasticity. Hagedorn and Manovskii (2008), for example, consider a shock to productivity, where the outside option does not change in response to this shock. Chodorow-Reich and Karabarbounis (2013) on the other hand, estimate the elasticity of the outside option with respect to a productivity shock to be close to one. We will refer to the former as a productivity shock, and the latter as a surplus shock.

Productivity Shocks

Under this specification, the elasticity is given by:[5]

$$\begin{array}{c}\text{elasticity of } \theta \\ \text{wrt } y\end{array} = \frac{\mathbb{E}[y]\overbrace{\left(\frac{\partial \mathbb{E}_z J_i\left(z', x, b\right)}{\partial y}\right)}^{\substack{\text{change in new} \\ \text{match value} \\ (>0)}}}{(1-\eta)\underbrace{\mathbb{E}_z J_i\left(z', x, b\right)}_{\substack{\text{current new} \\ \text{match value}}} - \theta\underbrace{\frac{\partial \mathbb{E}_z J_i\left(z', x, b\right)}{\partial \theta}}_{\substack{\text{change in new} \\ \text{match value} \\ (<0)}}} \tag{1.20}$$

where η is the elasticity of $f(\theta)$ with respect to θ.

Effect of costly negotiation. This equation illustrates why the various components of the model help increase observed elasticity. For example, consider a positive shock to productivity. Then

$$\frac{\partial J_i\left(z, x, \bar{w}\right)}{\partial y} = 1 - \frac{\partial w_i\left(z, x, \bar{w}\right)}{\partial y} + \beta\mathbb{E}_z\left[(1-s)\frac{\partial J_{i+1}\left(z', x+1, w_i\right)}{\partial y}\right] \tag{1.21}$$

[5] See Appendix A.1.2 for details

For the eldest generation, the last term drops out. Negotiation costs ensure that $\partial w_i/\partial y \leq \partial \tilde{w}/\partial y$ where $\tilde{w}$ is the wage that results from Nash Bargaining. Therefore, when firms and agents agree not to negotiate, the surplus goes to the firm. By solving backwards through generations, it is evident that the numerator of equation (1.20) increases.

The same analysis can be applied in the denominator. When there are no expected shocks, negotiation costs force the first term to be smaller than the case where there are no negotiation costs. For the second term, an increase in θ would cause wages to increase and firm surplus to decrease. However, negotiation costs reduce the expected wage change, and therefore reduce the loss of surplus. Thus, negotiation costs decrease the denominator.

Effect of Habit Formation. From equation (1.16), habit formation will have no effect when households have linear utilities. In the case of nonlinear utilities, as habits increase, so does $E - U$, which implies lower wages. This means wages respond less to a productivity shock than the case without habit formation and by equation (1.21), the change in surplus increases. Another way to think about this is that the surplus increase a worker gets from a productivity boost is tempered by decreasing surplus due to habit formation,[6] and firms are able to take advantage of this when negotiating wages.

Now consider the effect on expected firm surplus. Again, with linear utilities, habits will have no effect. With nonlinear utilities, however, J will be increasing in habit and therefore the age of the household. However, an increase in θ results in an expected increase in habit and reduces the response of wages. This implies that the change in firm surplus to an increase in match probability is less negative. The combined effect on the denominator is indeterminate. For large I, however, $\mathbb{E}J$ is close to the infinitely lived agent case, and therefore there is a net decrease in the denominator.

Effect of Lifecycle. Finally, let's consider the effect of the lifecycle model. If we remove other forms of heterogeneity, it is evident that the highest wage goes to the youngest worker. This does not, however, imply that firms hiring the youngest worker get the lowest surplus - in fact, it is exactly the opposite. Consider that firms have a cost to posting jobs. If they are "lucky" enough to hire a young worker, then they have the most time in which to recover that cost. With linear utilities, the elasticity can be solved analytically, and is given by:[7]

[6] I could also say they are increased by a decrease in habit due to unemployment, but due to asymmetry of probabilities of employment vs unemployment, the expected effect is a decrease in surplus due to habit.

[7] See Appendix A.1.3 for details

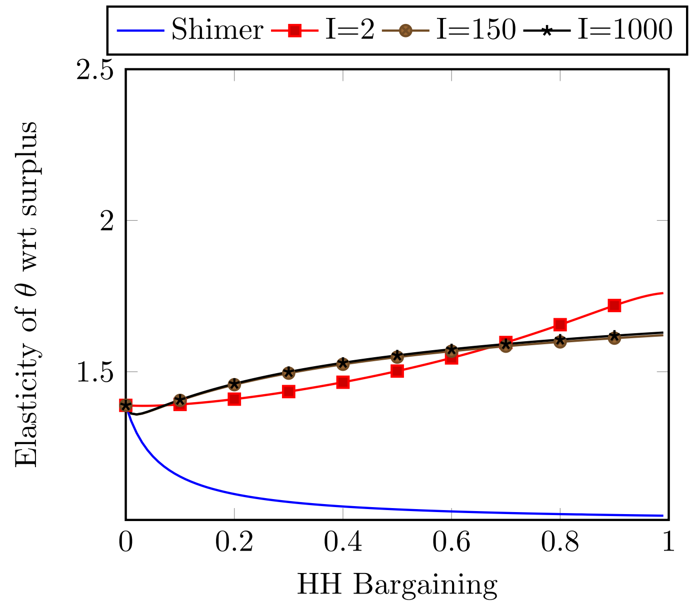


Figure 1.1: Each period is a month, RRA2

$$\frac{\mathbb{E}[y]}{\theta}\frac{d\theta}{d\left(\mathbb{E}[y]\right)} = \frac{g\left(x\right)}{\left(1-\eta\right)g\left(x\right)+\eta\gamma\beta f(\theta)\left(\frac{dg(x)}{dx}+2\frac{g(x)}{(1-x)}\right)}$$
$$g\left(x\right) = x^{I}+\left(I-2\right)\left(1-x\right)-2x+1$$
$$x = \beta\left(1-s-\gamma f\left(\theta\right)\right)$$

As $I \to \infty$, this converges to the elasticity given in Shimer (2005). With nonlinear utilities, the effect of generations depends on the negotiating power of household and their expected lifespan. To provide some intuition for this result. First consider the firm problem. Without a savings technology, households would prefer to have wages be as smooth as possible. In fact, basic decision theory tells us that households are willing to pay a premium to reduce this risk. Firms are aware of this, and offer lower wages to households, increasing firm surplus. Free entry then ensures that more firms enter the market, increasing the number of vacancies. As the number of periods a household lives increases, households are less willing to give up wages (the importance of any single period in the future is lower the more periods they live) as long as they have high enough wages over which to negotiate. Figure 1.1 shows the opposing effects of risk aversion and expected lifespan.

Surplus Shocks

If instead we test the surplus shock where the elasticity of b with respect to z equals one, then we have:

$$\begin{array}{c}\text{elasticity of } \theta \\ \text{wrt } y-b\end{array} = \frac{\mathbb{E}[y-b]\left(\frac{\partial \mathbb{E}_z J_i(z',x,b)}{\partial(y-b)}\right)}{(1-\eta)\,\mathbb{E}_z J_i\left(z',x,b\right) - \theta \frac{\partial \mathbb{E}_z J_i(z',x,b)}{\partial \theta}} \tag{1.22}$$

The analysis performed in the prior section holds even under the case of surplus shocks, though the expected elasticity is lower in magnitude. This is because the mechanisms described are not solely dependent on surplus gains due to unchanging outside options. Rather, they serve to effectively reduce the outside option, even when it is highly correlated with productivity shocks. That is, even though the outside option is procyclical, the mechanisms I include act as a friction to the perceived outside option, thereby allowing greater surplus to accumulate to the firm.

Parameterization.

The model has 12 parameters of interest: base productivity y, return to age r_a, home production b, elasticity of b with respect to z, worker bargaining power γ, discount rate r, cost of vacancy c, wage adjustment cost a, job finding rate $f(\theta)$, job separation rate s, support of habit, and the average number of shocks per month. Table 3.1 shows the values I select and the source. However, part of the exercise I undertake is to see the sensitivity of the elasticity with respect to these variables. Therefore, I will not pin down a, b, or γ.

The parameters are not intended as a serious calibration, but rather are intended to show the effects of the various mechanisms. As such, they are taken from a variety of sources without an attempt at matching moments in the data. A proper calibration is left for future work. To best address the issues raised in Chodorow-Reich and Karabarbounis (2013), I set ζ=1.

The following parameters are taken from Shimer (2005). $\beta = 0.996$ which implies a monthly interest rate of 0.004. The job finding rate is approximately 45% per month, the elasticity of job finding with respect to market tightness is 0.28, and the job separation rate is 3.4% per month. Shimer (2005) estimates that there are, on average, four aggregate shocks per quarter. As this is a monthly model, I assume the average monthly shock frequency is one third of this, i.e. 1.33. Additionally, as an approximation, I assume at most 15 shocks per month will occur.

Parameter	Value
β	0.996
Job finding rate, $f(\theta)$	0.45
Elasticity of f, η	0.28
Elasticity of b with respect to z, ζ	1
Job Separation rate, s	0.034
Base Production, y	1
Monthly return to age	0.001
Job Posting cost, c	0.005
Habit Support	$b/4$ to $3b/4$
Avg shocks per month	1.33

Table 1.1: Parameter values

As a negotiation cost is incurred upon hiring, having too high a posting cost eliminates much of the expected surplus from a match. I set a very low posting cost so that the various mechanisms can be seen.

I include a factor for increasing productivity with age. An easy explanation for this would be returns to tenure, though I don't explicitly model tenure. Using an increase of 0.001 per month, this implies that individuals gain approximately 10% productivity every ten years.

Results

Figures 1.2 and 1.3 illustrate the effects of home production and negotiation costs. Each band presents a level curve for the sum of home production and negotiation cost, increasing from 0.25 to 0.95. As this sum increases, so too does the elasticity of market tightness with respect to the surplus shock, consistent with Shimer (2005) and Hagedorn and Manovskii (2008). For the particular case illustrated, worker negotiation levels were set at 0.72 as per the Shimer (2005) calibration. As is apparent from the figures, the response of elasticity is non-linear in the two parameters of interest - even with linear household utility.

Comparing the two figures, we see that risk aversion significantly increases the response of market tightness to surplus shocks. Negotiation costs have a significant impact on expected wages, and losing one's job has two effects: a reduction in surplus due to negotiation when getting hired, and an increase in volatility of wages. The unequal response to risk between firms and workers shifts surplus gains to firms, and as θ moves with firm surplus, its volatility increases.

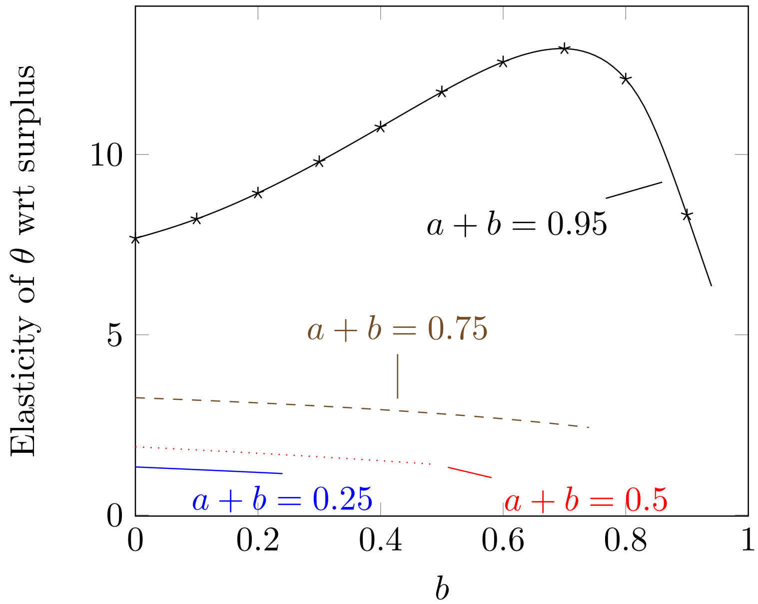


Figure 1.2: Linear Utility, $\gamma = 0.72$

Finally, figure 1.4 illustrates the impact of negotiating power. Considering the case of $a+b = 0.95$, the figure illustrates that the higher the negotiating power of the household, the lower is the elasticity (when considering linear household utilities). As household negotiating power decreases, surplus goes to the firm, and therefore the number of vacancies a firm posts due to a surplus shock is larger the greater is the firm's negotiating power.

Parameterizations of the model can also resolve the critique presented in Costain and Reiter (2008). For example, consider the parameterization where $a = 0.15$, $b = 0.7$ and $\gamma = 0.7$. The elasticity of θ with respect to a surplus shock is 5.4, which while lower than the data, is significantly higher than seen in the standard solutions. Costain and Reiter (2008) estimate the semi-elasticity of unemployment with respect to home production to be 3.09. In the context of this parameterization, I obtain a semi-elasticity of 4.1.

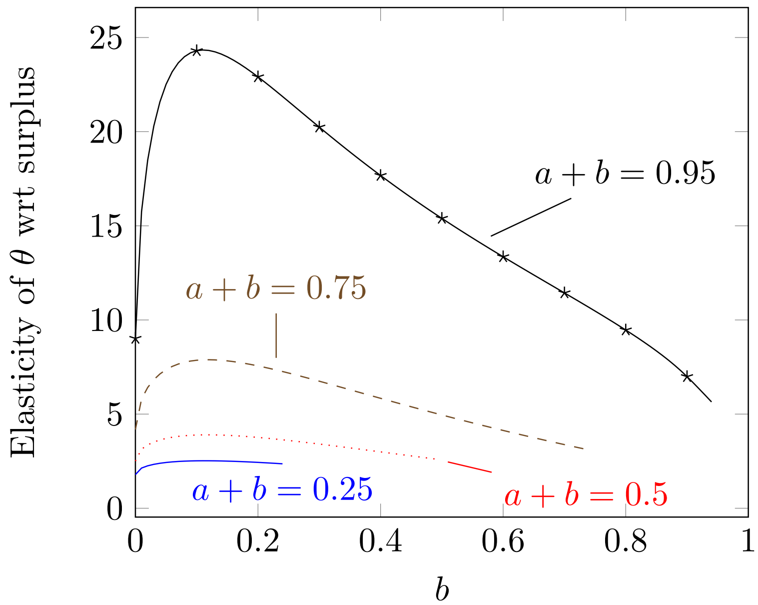


Figure 1.3: RRA2, $\gamma = 0.72$

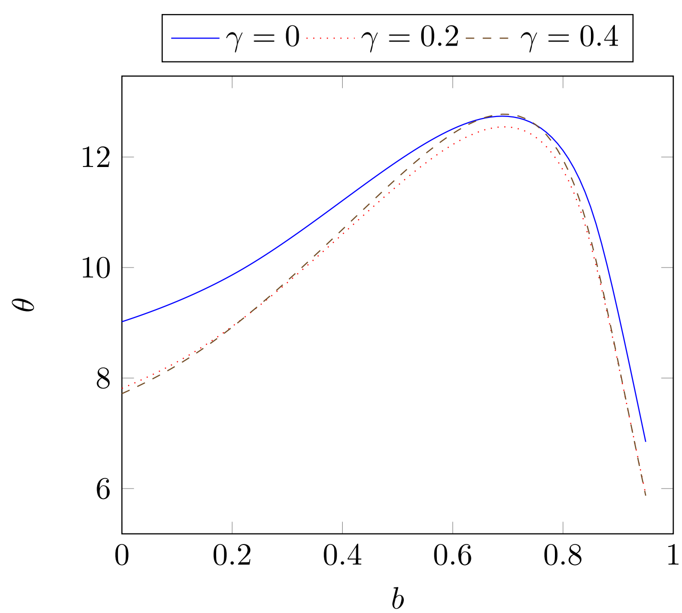


Figure 1.4: Linear Utility, $a + b = 0.95$

1.3 Model with endogenous labor market participation

The previous model identified different ways to affect the elasticity of θ with respect to y, the most effective method being to reduce the surplus of new hires. While previous works have accomplished this through implementing high outside options, I propose a model with labor choice and idiosyncratic productivity. This creates an extensive margin for workers, with a potentially non-trivial mass of workers entering and exiting the labor force in response to aggregate productivity shocks.

1.3.1 Environment

This model takes the standard Shimer model and adds worker skill heterogeneity via a one-time draw from a productivity distribution. The innovation is that this productivity draw, in combination with the aggregate productivity shock, can motivate workers to enter and leave the labor force. As will be shown, this movement on the extensive margin will have a large impact on job postings and measured unemployment.

The participation choice occurs as worker productivity is known by both workers and employers, with wages negotiated appropriately. Because the support of the productivity distribution allows for the existence of workers who would never be offered a wage greater than the outside option, these workers choose not to participate in the labor market. However, the majority of workers choose to participate based on the aggregate shock and distribution of current workers, which directly affect the wage offer.

The timeline of the model is as follows:

1. Households draw a permanent productivity level.
2. For each period,
   (a) Aggregate productivity shock, z, is realized
   (b) Exogenous separations occur, with probability s.
   (c) Firms post vacancies
   (d) Matching and Production take place simultaneously

For this model, we will also assume linear utilities.

Matching

Matching occurs exactly as in the previous model.

Worker Problem

Each household makes a one time draw from a productivity distribution, $G(i)$, which has a mean of zero and a support of $-\infty$ to ∞. To match previous notation, households will be indexed by i. Households with productivity draw i have output given by:

$$y_i(z) = e^{\tanh(i)} y(z)$$

We will let G be the standard normal distribution.

Workers now exist in one of three states; employed, unemployed, and not participating. Unemployed workers have a home production output of $b_i(z)$ that is a function of both their productivity and the aggregate productivity. Non-participatory workers receive an output of b^* that is independent of aggregate shock, but also obtain a leisure benefit of ϵ. Finally, employed workers get a negotiated wage, $w_i(z, \cdot)$. The value of the outside option is given by

$$b_i(z) = e^{tanh(i)} \zeta(z) b^*$$

Workers are subject to the same exogenous separation shock as in the previous model. As we will show, θ is now a function of the history of aggregate shocks, $\mathbf{z}$. The value of being unemployed to a worker of productivity i, is now given by:

$$\begin{aligned} U_i(\boldsymbol{z}) = b_i(z) + \beta \mathbb{E}_z \big[& \max(U_i(\boldsymbol{z'}), N_i(\boldsymbol{z'})) + \\ & f(\theta_{\boldsymbol{z'}}) \left\{ \overline{E-U}_i(\boldsymbol{z'}) \right\} \big] \end{aligned} \tag{1.23}$$

where $N_i(z)$ is the value of leaving the workforce, and

$$\overline{E-U}_i(\boldsymbol{z}) = \max(0, E_i(\boldsymbol{z}) - \max(U_i(\boldsymbol{z}), N_i(\boldsymbol{z}))) \tag{1.24}$$

We require (1.24) as this model allows the value of being employed to be less than that of being unemployed. This occurs because very unproductive workers produce close to zero output when there is a negative shock (see (1.28)). In this case, workers choose to reject a match when it occurs.

Separation can now occur in one of two ways, exogenously and endogenously. As such, we define the probability of separation, s_i as

$$s_i(\boldsymbol{z}) = \begin{cases} 1, & \text{if } E_i(\boldsymbol{z}) \le \max(U_i(\boldsymbol{z}), N_i(\boldsymbol{z})) \\ & \quad \text{or } J_i(\boldsymbol{z}) < V(\boldsymbol{z}) \\ s, & \text{otherwise} \end{cases}$$

The value of being employed to a worker of productivity i is now given by

$$\begin{aligned} E_i\,(z) = & w_i(\boldsymbol{z}) + \\ & \beta \mathbb{E}_{\boldsymbol{z}} \big\{ (1 - s_i(\boldsymbol{z'})) E_i\left(\boldsymbol{z'}\right) + s_i(\boldsymbol{z'}) \left\{ \max(U_i(\boldsymbol{z'}), N_i(\boldsymbol{z'}) \right\} \big\} \end{aligned} \tag{1.25}$$

When the expected value of employment is less than ϵ, workers choose to leave the labor force. In this case, the value of being out of the workforce, to a worker of productivity, i, is given by:

$$N_i\,(\boldsymbol{z}) = \bar{b}^* + \epsilon + \beta \mathbb{E}_z \left[\max\left(U_i(\boldsymbol{z'}), N_i(\boldsymbol{z'}) \right) \right] \tag{1.26}$$

We are interested in the productivity level where each of these curves intersect, as these points identify the productivity level at which a worker chooses to change their job search behavior.

In the previous model, workers never chose to leave a match, as the value of employment weakly dominated that of being unemployed. In this model, offered wages could be less than the outside option due to low productivity and negative shocks. With a sufficiently negative productivity shock, employed workers could choose to leave their job.

Proposition 1. *$\exists i^*_{eu}$ such that $U_{i^*_{eu}}(\boldsymbol{z}) = E_{i^*_{eu}}(\boldsymbol{z})$*

Proof. Assume the support of G is such that $y_{\underline{i}}(z) < b_{\underline{i}}(z)\ \forall z \in Z$, where $\underline{i}$ is the lowest possible idiosyncratic productivity. As wages will be less than the outside option, if we assume that the transition process between aggregate shocks is such that $E_{\underline{i}}(\boldsymbol{z'}) < U_{\underline{i}}(\boldsymbol{z'})$, it is evident that $E_{\underline{i}}(\boldsymbol{z}) < U_{\underline{i}}(\boldsymbol{z})$. As idiosyncratic productivity increases, output increases until it surpasses the outside option. At this point, Nash Bargaining allocates some of the surplus to the worker, which is reflected in wages. As such, the first term of (1.25) is greater than the first term of (1.23).

As the point of this crossover, the value of $E_i(\boldsymbol{z})$ is still less than $U_i(\boldsymbol{z})$. As such, the second term in (1.25) is less than the second term in (1.23). However, this second term is increasing in i in (1.25) but equal to 0 in (1.23). As productivity increases, then, $E_i(\boldsymbol{z})$ is increasing faster than $U_i(\boldsymbol{z})$. Therefore, as long as the support of G is sufficiently broad, $\exists i^*_{eu} \in G s.t. E_{i^*_{eu}}(\boldsymbol{z}) = U_{i^*_{eu}}(\boldsymbol{z})$ □

Proposition 2. *$\exists i^*_{un}$ such that $U_{i^*_{un}}(\boldsymbol{z}) = N_{i^*_{un}}(\boldsymbol{z})$.*

Proof. It is evident that at $\underline{i}$, $U_{\underline{i}}(\boldsymbol{z}) < N_{\underline{i}}(\boldsymbol{z})$. While this holds, N_i is constant in i but U_i is increasing in i. For above average productivity workers, $b_i(z) > b^*$. From **??** , $\exists i \in G$ such that $U_i < E_i$, which implies that, with sufficient support, $\exists i \in G$ such that $f(\theta_{\boldsymbol{z'}})\left\{\overline{E - U}_i(\boldsymbol{z'})\right\} > 0$. As U and N are continuous, it must be that $\exists i^*_{un} \in G$ such that $U_{i^*_{un}}(z) = N_{i^*_{un}}(z)$. □

Proposition 3. *$\forall i > i^*_{eu}$, $E_i(\boldsymbol{z}) > U_i(\boldsymbol{z})$*

Proof.

$$\begin{aligned}\Delta(E_i(\boldsymbol{z}) - U_i(\boldsymbol{z})) &= \Delta(w_i((\boldsymbol{z})) - b_i(\boldsymbol{z})) + \beta\Delta\left\{\alpha(E_i(\boldsymbol{z'}) - U_i(\boldsymbol{z'}))\right\} \\ &= \sum_{t=0}^{\infty} \beta^t \Delta(w_i((\boldsymbol{z})) - b_i(\boldsymbol{z}))\end{aligned} \tag{1.27}$$

As wages split the surplus, the difference in 1.27 is always positive. Therefore, the summation is positive, and so, $\forall i > i^*_{eu}$, the difference is positive. □

Proposition 4. *$\forall i > i^*_{un}$, $U_i(\boldsymbol{z}) > N_i(\boldsymbol{z})$*

Proof. Obvious □

Proposition 5. *$\exists i^*_{en}$ such that $E_{i^*_{en}}(\boldsymbol{z}) = N_{i^*_{en}}(\boldsymbol{z})$.*

Proof. From Proposition 1, at $\underline{i}$, $E_{\underline{i}}(\boldsymbol{z}) < N_{\underline{i}}(\boldsymbol{z})$. From Proposition 3 and Proposition 4, $\exists i \in G$ such that

$$E_i(\boldsymbol{z}) > U_i(\boldsymbol{z}) > N_i(\boldsymbol{z})$$

As E, U, and N are all continuous, it must be that $\exists i^*_{en}$ such that $E_{i^*_{en}}(\boldsymbol{z}) = N_{i^*_{en}}(\boldsymbol{z})$. □

The implications of these propositions is that for a given aggregate productivity level, z, there exists a cutoff idiosyncratic productivity for workers where, in response to an aggregate shock, they may choose to either leave their job or the workforce entirely. Note that this only occurs with a negative productivity shock. Positive shocks increase the value of employment, and therefore reinforce the choice to be employed.

Firm Problem

As before, all firms have a single position and access to the same exogenous production technology. The productivity, $y_i(z)$, of a match is dependent on worker type, i, and the aggregate shock, z, and is given by

$$y_i(z) = e^{tanh(i)}(b^* + e^z(y^* - b)) \tag{1.28}$$

where y^* is the long-run average output of the agent with mean productivity and b^* is the long run, average, outside option.

Firms are assumed to be risk neutral, so the value to a firm matched with an agent of type i with negotiated wage, $\bar{w}$, is

$$\begin{aligned} J_i(\boldsymbol{z}) \quad &= \quad \max\{V(\boldsymbol{z}), y_i(\boldsymbol{z}) - w_i(\boldsymbol{z}) + \\ &\qquad \beta\mathbb{E}_z\left[s_i(\boldsymbol{z'})V(\boldsymbol{z}) + \left(1 - s_i(\boldsymbol{z'})\right)J_i(\boldsymbol{z})\right]\} \end{aligned} \tag{1.29}$$

When the position in the firm is vacant, the value to the firm is given by

$$V(\boldsymbol{z}) \quad = \quad -c + \tag{1.30}$$

$$\beta \int_{\boldsymbol{z}'} \left\{ q(\theta_{\boldsymbol{z}}) \int_{i > i^*_{un}(\boldsymbol{z'})} g(i|\boldsymbol{z'})J_i(\boldsymbol{z'})di + (1 - q(\theta_{\boldsymbol{z}}))V(\boldsymbol{z'}) \right\} d(z'|\boldsymbol{z}) \tag{1.31}$$

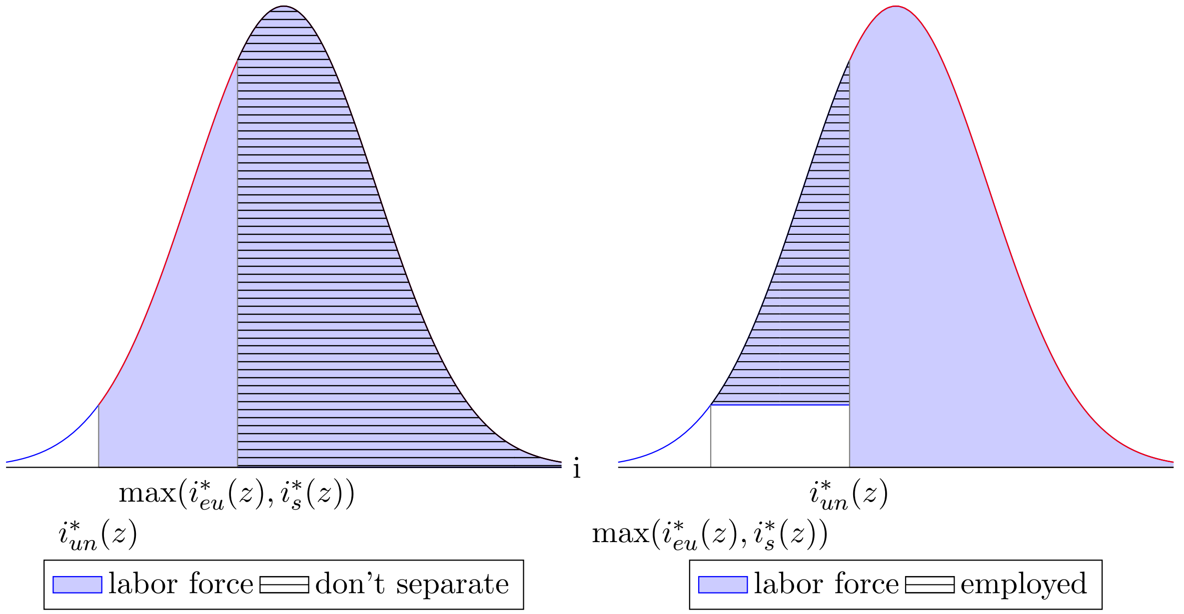
Note the inner integral, $\int_{i>i^*_{un}(\boldsymbol{z'})} g(i|\boldsymbol{z'})J_i(\boldsymbol{z'})di$. The value of a vacancy is a function of individuals who are searching for a job, not all workers without jobs as is usually the case. As discussed, the workers on the margin between being in the labor force and out have lower productivity than those who choose to move from employed to unemployed, and therefore, we integrate from $i^*_{un}(\boldsymbol{z'})$ as opposed to $i^*_{eu}(\boldsymbol{z'})$.

As with workers, firms may choose to separate from a low productivity worker when there is a negative shock causing $J_i(\boldsymbol{z}) < V(\boldsymbol{z})$. We will denote by i^*_s the productivity level that sets $J_{i^*_s}(\boldsymbol{z}) = V(\boldsymbol{z})$.

Finally, free entry of firms results in

$$V(\boldsymbol{z}) = 0, \forall \boldsymbol{z} \tag{1.32}$$

Figure 1.5 illustrates the relationship between individual productivity and labor force status for a given shock level. In 1.5a, there exist a mass of workers who were employed but



(a) $i^*_{un}(z) < \max(i^*_{eu}(z), i^*_s(z))$ (b) $i^*_{un}(z) > \max(i^*_{eu}(z), i^*_s(z))$

Figure 1.5: labor force composition

have quit their jobs (or been let go). In 1.5b, employed workers remain in the workforce, but unemployed workers leave the workforce.

Shock Process

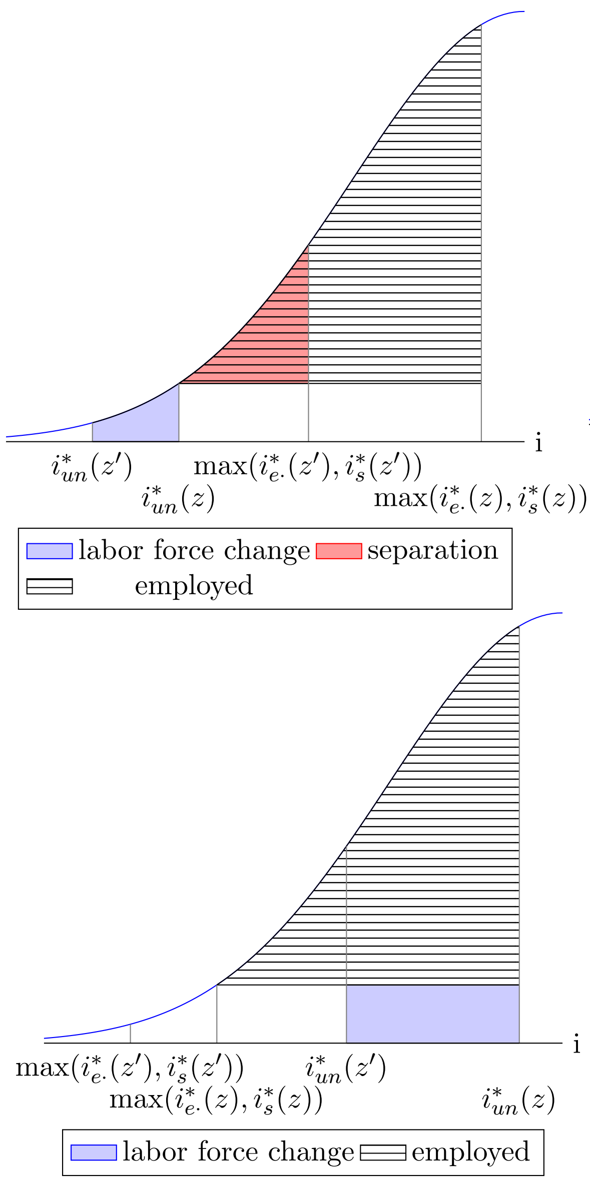
The shock process does not change from the previously discussed model.

Unemployment Transition

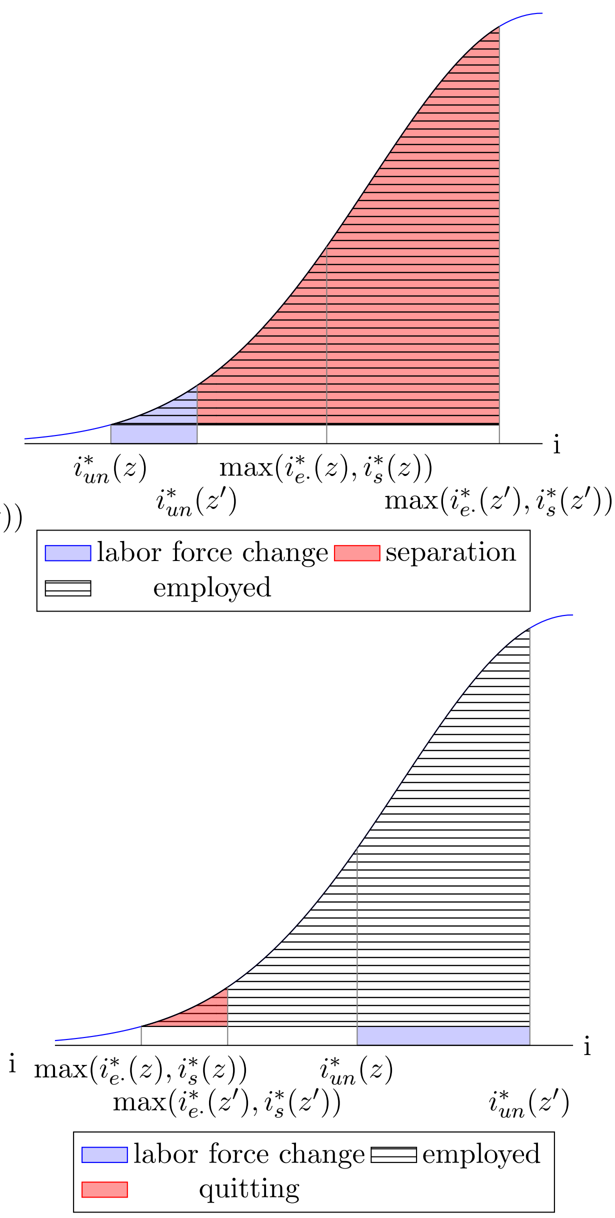
As in the previous model, the number of individuals who are unemployed is not equal to the number of people who are job hunting. In this version, we must add/subtract those workers who enter/exit the labor force to those searching previously. Additionally, as unemployment is generally measured relative to participation, we must also modify the definition previously considered to adjust for the measure of individuals who are not in the labor force.

Figure 1.6a illustrates the effect of a positive aggregate shock. Specifically, there is movement on the extensive margin of job hunters with individuals entering the labor force and a reduction in the productivity cutoff for voluntary separations. Note that there may exist individuals who are in the labor force while they are employed, but will leave the labor force if they become unemployed.[8] Similarly, figure 1.6b illustrates the situation where

[8] Bottom graph in 1.6a



(a) Effect of positive shock



(b) Effect of negative shock

Figure 1.6: Effect of shocks

there is a negative aggregate shock. In this case, we have workers becoming discouraged and leaving the labor force, with some even quitting jobs to do so.

If we let N and L be the mass of individuals, prior to the shock, outside and inside the labor force, respectively. Then, obviously, N+L=1. Let u be the unemployment rate (as a percentage of labor force) and $e(i)$ be the percentage of individuals of productivity i that are unemployed. Assuming a positive shock, the new labor force level is

$$L' = L + \int_{i^*_{un}(z')}^{i^*_{un}(z)} (1 - e(i))di$$

The measure of voluntary separations into unemployment is

$$s_v = \max(0, \int_{\max(i^*_{un}(z), i^*_{un}(z'))}^{\max(i^*_{e\cdot}(z'), i^*_s(z'))} e(i)di)$$

The measure of employed quitting out of the labor force is

$$s_n = \max(0, \int_{\min(i^*_{un}(z), \max i^*_{e\cdot}(z), i^*_s(z))}^{\max(i^*_{e\cdot}(z'), i^*_s(z'))} e(i)di)$$

The measure of individuals searching for a job is given by

$$u_h = uL + (L' - L) + s_v + s \int_{\max(i^*_{un}(z'), i^*_{e\cdot}(z), i^*_s(z))} e(i)di$$

market tightness is given by

$$\theta = \frac{v}{u_h}$$

and the new unemployment rate entering the next period is given by

$$u' = \frac{(1 - f(\theta))\, u_h}{L'}$$

Note that the denominator for market tightness in this model is u_h and not u.

The change in unemployment is a function of the distribution of unemployment amongst households. As such, there isn't a one-to-one mapping between θ and shocks. Rather, if Ω is the distribution of unemployment, we have $u : (\Omega, z) \rightarrow \mathbb{R}$.

Wage Determination

Wages are negotiated as before.

1.3.2 Equilibrium

Characterization. Let $\mathbf{z}$ be the history of shocks and s be the exogenous separation rate. Then, given s and $\theta(\mathbf{z})$, an equilibrium is a wage profile, $\mathbf{w}$, and vacancy profile, $\mathbf{v}$, such that for all $i \in I$, $\boldsymbol{z} \in \boldsymbol{Z}$, $w \in \mathbf{w}$, equations (1.23), (1.25), (1.26), (1.28), (1.29), (1.31), (1.32), and (1.13) are satisfied.

The solution to Nash Bargaining is the key equation for wages, which are set such that the appropriate solution holds with equality whenever wages change. To simplify notation, we will define the function $\bar{U}_i(z) = \max(U_i(z), N_i(z))$ and solve as before.

As before, wages can be uniquely determined, and are given by:[9]

$$w_i\left(z\right) = \left(1-\gamma\right)b + \gamma y_i\left(z\right) + \gamma\beta\mathbb{E}_z\left\{f\left(\theta_z\right)J_i\left(z'\right)\right\} \tag{1.33}$$

That is, firms pay to workers a portion of the benefit they get by the worker not quitting every period.

Solution. (1.31), along with the free entry condition (1.32), provides the equality that must hold for any aggregate shock. With wages given by equation (1.33), for any $\theta(z)$, each of the terms in (1.31) can be solved.

1.3.3 Comparative Statics

The primary challenge of this model is that the elasticity if a function of the distribution of unemployment amongst all productivity levels. This prevents a general solving of the value function using standard methods. However, to determine if the general mechanism generates sufficient movement, we can evaluate the elasticity of θ with respect to a shock in a static environment. To do so, we again restate equation (1.31) as

$$\frac{c}{\beta} = \frac{f\left(\theta_{\boldsymbol{z}}\right)}{\theta_{\boldsymbol{z}}}\mathbb{E}\left\{\int_{i>i^*_{un}(z')} g(i|\boldsymbol{z'})J_i(\boldsymbol{z'})di\right\}$$

and take its derivative with respect to the appropriate type of shock.

[9]See Appendix A.2.1 for details

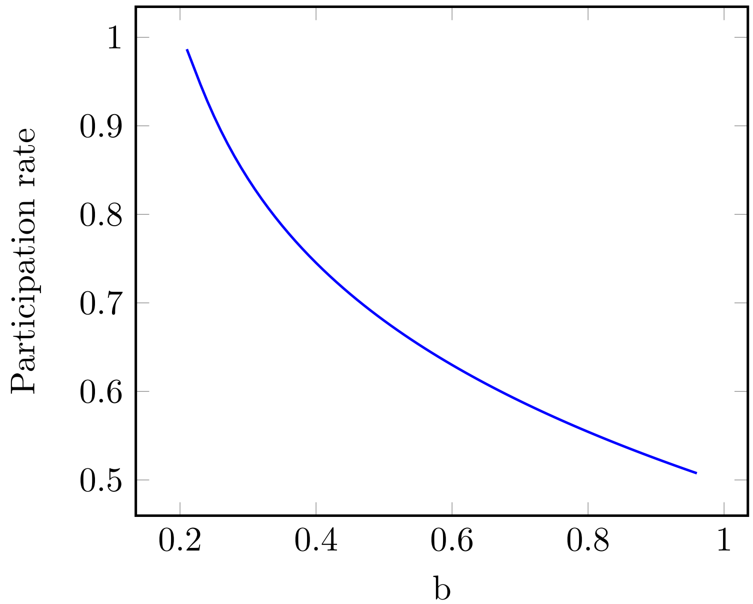


Figure 1.7: Effect of b on participation

Parameterization

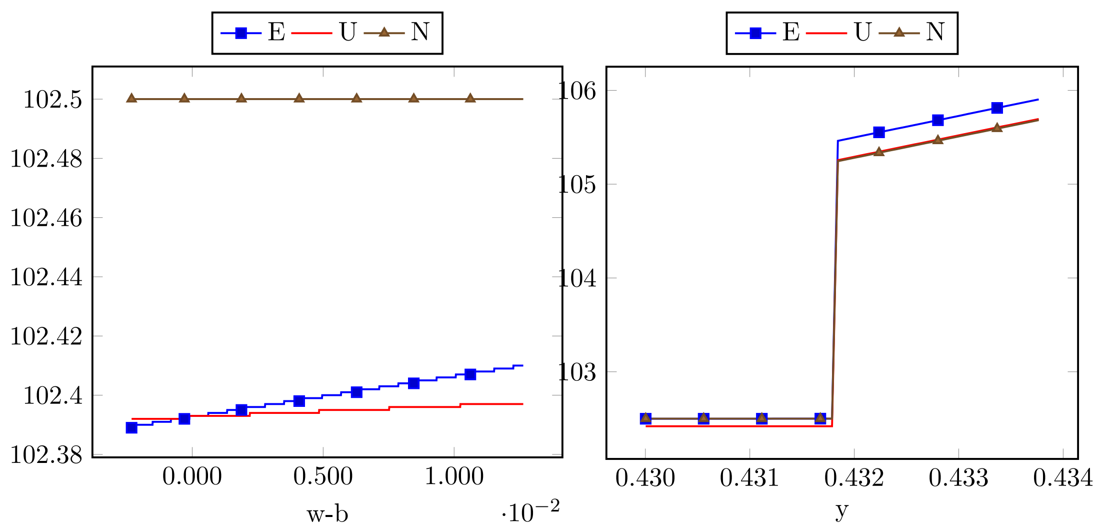
The model has eleven parameters of interest: base productivity y, home production b, elasticity of b with respect to z, disutility of working ϵ, worker bargaining power γ, discount rate r, cost of vacancy c, job finding rate $f(\theta)$, job separation rate s, productivity distribution g, and the average number of shocks per month. Table 1.2 shows the values I choose as defaults.

As before, the parameters are not intended as a serious calibration, but rather are intended to show the effects of mechanism discussed. One consideration we now must make is to try to match the participation rate, which is largely dependent on ϵ and b. The effect on participation rates of varying b while holding other parameters constant is illustrated in figure 1.7.

Figure 1.8 illustrates the worker cutoffs where we assume that there are no aggregate shocks. As would be expected, the participation crossover, i^*_{un} occurs near to where the wage equals the outside option. Exact equality is not required, as with multiple states, a worker may be willing to take a lower wage today in order to obtain higher wages tomorrow. As expected, there is a region where employed workers continue to work, but unemployed workers leave the workforce.

Parameter	Value
β	0.996
Job finding rate, $f(\theta)$	0.45
Elasticity of f, η	0.28
Elasticity of b with respect to z	0
Job Separation rate, s	0.034
Base Production, y	1
Outside option, b	0.4
Disutility of job search, ϵ	0.01
Job Posting cost, c	0.005
Productivity distribution, g	$N \sim (0,1)$
Avg shocks per month	1.33

Table 1.2: Parameter values



(a) i^*_{eu}

(b) i^*_{un}

Figure 1.8: Cutoffs

Productivity Shocks

The equation for the elasticity of θ with respect to y is very similar to that obtained previously, with the primary difference being the underlying distribution of job seekers. Whereas in the previous model all households actively searched for employment, in this model, only those households whose productivity is sufficiently high will do so. The equation for elasticity is, however, very similar, being

$$\begin{array}{c}\text{elasticity of } \theta \\ \text{wrt } y\end{array} = \frac{\mathbb{E}[y] \overbrace{\left(\frac{\partial \mathbb{E}\left[\mathbb{E}_{z,z'} J_i(z')\right]}{\partial y} \right)}^{\substack{\text{change in new} \\ \text{match value} \\ (>0)}}}{(1-\eta) \underbrace{\mathbb{E}\left[\mathbb{E}_{z,z'} J_i\left(z'\right)\right]}_{\substack{\text{current new} \\ \text{match value}}} - \theta \underbrace{\frac{\partial \mathbb{E}\left[\mathbb{E}_{z,z'} J_i(z')\right]}{\partial \theta}}_{\substack{\text{change in new} \\ \text{match value} \\ (<0)}}} \tag{1.34}$$

where η is the elasticity of $f(\theta)$ with respect to θ.

Comparing this to (1.20), we notice that the main difference is in the expected value of J. Consider the following example.

Example. Consider a steady state where $L = 0.9$, and $u = 0.05$. Now consider a shock where $\Delta L = 0.01$. An additional 1% of the population is added to the 4.5% of the population that is unemployed. That is, approximately 18% of those now unemployed have extremely low productivity, significantly decreasing the expected value of J. This has the effect of increasing the numerator and decreasing the denominator, and so we would expect an increase in the elasticity.

Another significant difference is that the elasticity is a function of the unemployment levels by productivity, itself a function of the history of shocks. As such, this model breaks the connection between shock and response, requiring multiple periods for the model to completely adjust to a shock rather than the single period in Shimer (2005).

Figure 1.9 shows the elasticity of θ with respect to a positive shock in the period of the shock, where the x-axis allows for the varying of bargaining power. The elasticity in the second period is very close to that in the first period, but in the opposite direction. The combination of these two periods yields an elasticity that is of a similar order of magnitude as the results in Shimer (2005), but not exactly equal due to history dependence.

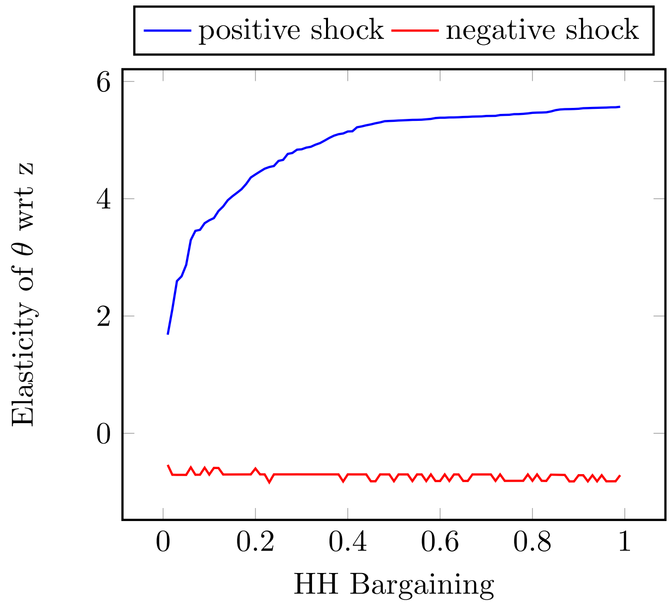


Figure 1.9: Effect of HH Bargaining

This elasticity is not symmetric, in that negative shocks yield significantly different elasticities. This is to be expected, as the extensive margin does not add to the mass of unemployed workers searching for jobs in the first period. Figure 1.9 also illustrates this elasticity, which is of the same order of magnitude as that in Shimer (2005). However, as there is still an impact on the exogenous margin, we would not expect equality of outcomes.

These results are also sensitive to the value of the outside option, but in the opposite direction as that presented in Hagedorn and Manovskii (2008). As seen in Figure 1.10, decreasing the outside option actually increases the observed elasticity. Mechanically, with low outside options, the individuals choosing not to work are those with very low productivity. As a result, the value of new matches are extremely low when a shock occurs, increasing the observed elasticity. This result is discontinuous at the limit where b is less than the output produced by the least productive worker. At this point, we simply have the Shimer outcome.

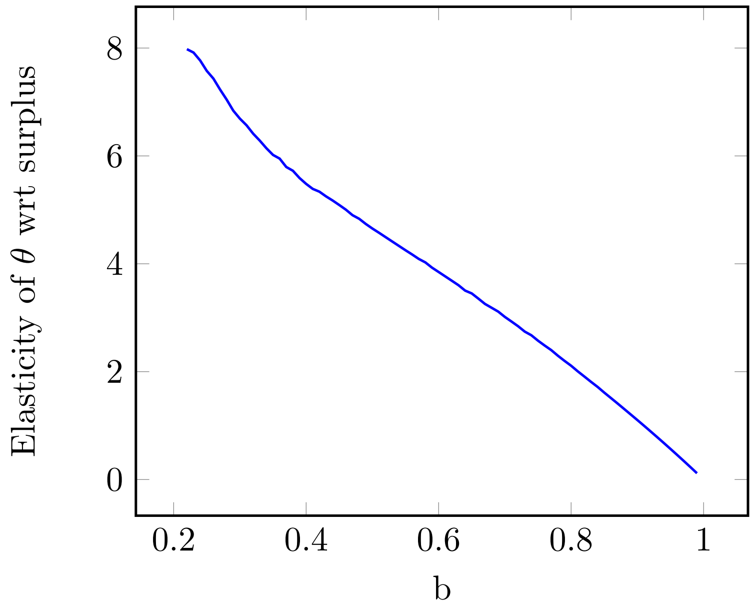


Figure 1.10: Effect of b

Surplus Shocks

Once again, we can examine how the results change when the impact of the aggregate shock affects the outside option.

$$\begin{matrix}\text{elasticity of } \theta \\ \text{wrt } y-b\end{matrix} = \frac{\mathbb{E}[y-b]\overbrace{\left(\frac{\partial \mathbb{E}\left[\mathbb{E}_{z,z'} J_i(z')\right]}{\partial (y-b)}\right)}^{\substack{\text{change in new} \\ \text{match value} \\ (>0)}}}{(1-\eta)\underbrace{\mathbb{E}\left[\mathbb{E}_{z,z'} J_i\left(z'\right)\right]}_{\substack{\text{current new} \\ \text{match value}}} - \theta\underbrace{\frac{\partial \mathbb{E}\left[\mathbb{E}_{z,z'} J_i(z')\right]}{\partial \theta}}_{\substack{\text{change in new} \\ \text{match value} \\ (<0)}}} \tag{1.35}$$

The previous results are equivalent to having an elasticity of b with respect to z equal to 0. Figure 1.11 plots the elasticity of θ with respect to a shock using the previously specified parameters while varying this elasticity.

This model yields results that move in the opposite direction of those in Chodorow-Reich and Karabarbounis (2013). Specifically, increasing the correlation between the outside option and aggregate shocks serves to *increase* the elasticity. The primary driver for this is with the outside option of the non-participating workers - which is independent of the idiosyncratic productivity level. As a result, a shock to aggregate productivity affects the

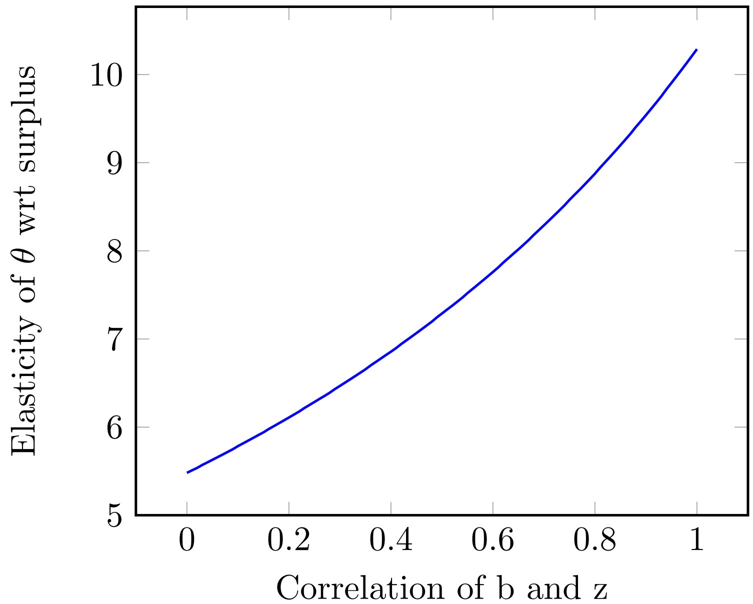


Figure 1.11: Effect of correlation between b and z

participation choice for households differently, as well as the expected surplus for firms. For the participation levels in this parameterization, the increased benefit of working outweighs the gain in utility for not working. As a result, more low-productivity workers enter the labor market, causing a larger change in θ.

1.4 Conclusions

This paper introduces several novel approaches to solving the Shimer puzzle. In the first model, it introduces a cost to bargaining which agents have the option to avoid by not renegotiating wages in a period. This slight change results in significant increases in the elasticity of market-tightness with respect to both productivity and surplus shocks. Heterogeneity in the form of lifecycle and habit formation are then added, which allow the model to better match the data.

This model is quite tractable, and allows for easy inclusion of additional assumptions. Recent work has shown that separation rates are not exogenous, either with respect to an individual or the aggregate economy. This model and its associated computation can easily be extended to include endogenous separation rates conditional on type. Alternatively, we could drop the assumption of perfect information, and consider informational costs associated with determining agent type.

The second model introduces skill heterogeneity and an extensive margin for the labor decision. The entry and exit of low productivity workers in response to aggregate shocks has a significant impact on the expected utility of new hires, and therefore the elasticity of θ with respect to these shocks. The primary issue with this model, however, is that it is dependent on the distribution of unemployment, which makes providing a general solution extremely difficult. However, when examining a static model, it obtains elaticities of market tightness similar to that found in the data, indicating that the exogenous margin may be important when attempting to model the labor market.

There are several potential avenues to solving this model in future research. For example, a model with limited foresight could be used solved while simulating an unemployment path to obtain a series for unemployment and vacancies. This would prevent the need to solve the model for the entire distribution, requiring rather to solve for a specific path. As shocks follow a random process, one can solve for as long a time series as desired. This solution would require a significant amount of computing power, though, as it would need to solve the entire model for each period in the series.

That worker heterogeneity provides a promising avenue to obtain a solution to the Shimer Puzzle is not surprising by itself, as there has been quite a bit of research in that area. However, this paper introduces several models that are potentially simpler to implement than previous solutions, and with some further research, should provide a model that can be easily integrated into other work.

Chapter 2

Asymmetric International Transmission [1]

2.1 Introduction

The Financial Crisis of 2008 originated in the United States and spread globally. While it started in the United States, the crisis did not actually hurt the United States the most, nor were all countries affected equally. When compared to the Scandinavian crisis in the early 1990s or the Mexican Peso Crisis in 1995, The 2008 crisis is markedly different in its global reach. In the case of the Mexican crisis, the impact was felt widely throughout Latin America, but the U.S. was unaffected. This paper looks at how financial crises spread and argues that the relative degree of financial market sophistication explains both the varying responses to and ability to cause financial crises. The degree to which financial markets are developed determines the ability of an economy to insure against non-financial shocks, which in turn determines the desire to hold risky capital assets relative to safe debt instruments. This leads to financially well-developed countries holding large shares of capital assets both domestically and abroad. Then, when a financial crisis hits, this affects the ability of the financial system to insure non-financial shocks, and, in response, capital holdings decrease, which transmits the crisis from the well-developed economy to the lesser-developed economies.

Using the capital flow database started by Lane and Milesi-Ferreti (2005), as well as data from the U.S. Treasury International Capital (TIC) database, we are able to examine capital flow patterns to show that financial flows in general, as well as flows from the United

[1] Co-Authored with Rob Mrkonich

States specifically, are strongly correlated with output growth and are of an economically significant magnitude. We document significantly decreased capital flows during the 2008 financial crisis in not only the U.S. but other financially-developed economies as well. Using the aforementioned databases, we run preliminary regressions to establish some basic correlations between financial flow type and output and find that productive capital flows such as portfolio investment show a strong positive coefficient that is of economically significant magnitude. From this we hypothesize that financial development is key to both explaining the types of flows countries hold, as well as the transmission of the shocks.

The model used in this paper is most closely related to Mendoza et al. (2009). As in their paper, we allow for varying completeness of markets by imposing a restriction on the selling of state contingent bonds. Conveniently, this is equivalent to having only an uncontingent bond and transforming the idiosyncratic shocks faced by households in the economy into a linear combination of the raw shock value and its conditional expectation. In that sense, it is easy to see the main mechanism of the model: more developed financial systems translate directly into dampening the variance of the shocks faced in that economy. Thus, when a financial crisis distorts this ability, households now face more variance than before and, if they are risk averse, hold less capital. The more dependent on foreign capital an economy is, the stronger the contagion. As financially well-developed countries are not dependent on capital from less developed economies, the transmission of a financial crisis is fundamentally reliant on the country of origination and leads to asymmetric transmission.

The main contribution of this paper is to provide a model of international transmission in which the countries are not symmetric. This is, to our knowledge, the first model of transmission where no country is assumed to be a small open economy that generates such a result. This is important as many financial crises occur in countries that are not only financially important, but also economically large, making the assumption of a small open economy untenable.

Related Literature:

This paper fits into two main literatures, namely the literature on capital flows and, more specifically, global imbalances, as well as the literature on international transmission and contagion. The main mechanism in our paper is reliant upon the composition of capital flows, specifically on the idea that more financially developed countries hold larger positions in capital investments, such as foreign portfolio investment or foreign direct investment, as opposed to debt instruments. This ties in directly with the current literature on global imbalances which looks at the large purchases of U.S. debt instruments by foreign economies and is summarized thoroughly in Gourinchas and Rey (2014). Lane and

Milesi-Ferreti (2005), in addition to creating the aforementioned database, establish the basic composition of flows, while Lane and Milesi-Ferretti (2014) discusses how flows have changed after the 2008 Financial Crisis. In that same vein, Milesi-Ferretti et al. (2010) examines bilateral linkages and finds limited importance of the size of gross vs. net external positions in explaining the transmission of the crisis. Our paper extends this and looks into the importance of composition of flows and finds instead that the size of risky capital flows is important. Along these lines, Fogli and Perri (2015) finds evidence that, in OECD countries, the precautionary savings motive is key to explaining their net foreign asset position.

Finally, from a modeling perspective, our paper closely follows Mendoza et al. (2009), which argues that differences in financial development help to explain trends in capital flows. They show that debt instruments represent a safe investment, while capital holdings, which are riskier, are held by those better able to bear the risk. As it reproduces the capital flows observed in the data, we use their model as a basis for ours and extend it to allow for examining contagion as well. Their paper is related to the work done in Caballero and Farhi (ated) and Caballero and Farhi (2013), which emphasizes the importance of U.S. debt as a safe asset and examines the policy consequences of a shortage of safe assets.

After the 2008 Financial Crisis many papers were published on the importance of financial linkages in spreading recessions. In most models, most economies are symmetric, meaning that, regardless of where the origination of the shock occurred, there is an effect on the other country. The only asymmetry that can exist is in the severity of the negative effect. In terms of mechanisms for transmission, there are two common avenues. First, having segmented financial markets can form a path for transmission. In Kalemli-Ozcan et al. (2013) some firms receive funding from international banks, which then are the vehicle for transmitting the negative shock. Second, collateral constraints are used by many papers to Perri and Quadrini (2011) use a model with collateral constraints where the value of the collateral is endogenous. While these papers have symmetric outcomes, Acemoglu et al. (2013) examines systematic risk and transmission using network theory, which, while not explicitly done in that particular paper, has the ability to make contagion starting node dependent. Our model is a large open economy model studying contagion. A paper with a similar scope is Aysun and Alpanda (2012) which uses a large open economy DSGE model with credit market frictions to examine the financial crisis, and finds that the baseline model cannot match the data. When adding in a global financial sector, contagion is strengthened significantly, though only when there exists friction in the international financial contracts.

Our model features a portfolio choice along the lines of both Devereux and Yu (2014) and Devereux and Yetman (2010), though, by introducing varying completeness of financial markets, our results are very different from theirs. Devereux and Yetman (2010) finds that financial linkages alone cannot produce transmission, and that leverage constraints are key in generating transmission. In addition, they find trading debt instruments only is insufficient for transmission, meaning that equity plays an important role. In Devereux and Yu (2014), financial linkages increase the occurrence but decrease the severity of crises. In our paper, the type of, not simply the existence of, financial linkages affects the incidence of transmission. Also, the impact of a crisis is related to the type of capital flow, meaning specifically that countries that are more dependent upon foreign financing of capital suffer from stronger contagion. These results are more in line with Amaral and Quintin (2010), who find that differences in financial development lead to large effects on output. Amaral and Quintin (2010) generates varying financial development through limited commitment, and we do so similarly by modeling financial development in a way that is equivalent to an economy with limited enforcement of contingent contracts.

From a data perspective, the importance of financial constraints is documented by Campello et al. (2009) who examine the results of a survey of over 1000 CFOs from across the U.S., Europe, and Asia. They find that credit constraints inhibited financing quality projects, both in the U.S. and abroad, and in many cases the results were amplified in Europe and Asia. This information is important to incorporate in our model, which we do through both our limited liability constraint as well as the constraint limiting the ability of bonds to span the idiosyncratic shocks present in the economy.

Most papers on contagion in the wake of the 2008 crisis focus on the role of banks in propagating the negative shock across borders. Meh and Moran (2008) focus on the importance of what they term the bank capital channel. In their model, bank capital helps to alleviate agency problems, which, when there is not enough bank capital, leads to insufficient loanable funds and macroeconomic misallocation that amplifies negative shocks. Other studies such as Kenourgios et al. (2011), which focuses on the interactions of two major financial centers, the U.S. and U.K., with four main emerging markets, Brazil, Russia, India, and China, during five recent financial crises, lend support to our claim that international transmission is asymmetric. They find that the emerging economies suffer more frequently from contagion than the U.S. or U.K.

The remainder of the paper is structured as follows: Section 2 provides insight into capital flows and the impact of the financial crisis across borders. Section 3 introduces the model and characterizes its solution. Section 4 details the parameterization of the model. Section 5 provides the main results. Section 6 briefly concludes.

2.2 Data

This section has two objectives: First, to document the large changes in U.S. capital flows following the financial crisis in 2008, and, second, to demonstrate the importance of foreign capital flows to capital formation and observed output abroad. To do this, two key data sources are used. First, Philip Lane and Gian Maria Milesi-Ferretti maintain a database of financial flows from 1970 to 2011 of over 180 countries. The database provides information on aggregated holdings as well as the composition of the holdings (debt, portfolio investment, direct investment, reserves, etc.). The second dataset is the U.S. Treasury International Capital (TIC) database, which provides data on U.S. financial holdings by country. Specifically, it tracks both which foreign countries hold U.S. securities and U.S. holdings of foreign securities by country, and it also includes information on both aggregate holdings as well as their composition. The Lane-Milesi-Ferretti database is useful for showing the general importance of financial flows, while the TIC database will be used to show the importance of U.S. capital flows to other countries.

Figure 2.1 shows how the U.S. balance sheet has changed over time. A number of things call for notice. Starting in 1980 and going through 2011, both U.S. asset holdings and liabilities have grown quickly and are now multiple orders of magnitude larger. As of 2011, financial flows are almost double the size of GDP after having previously been a small fraction. This is the fundamental motivation behind our project: in previous recessions, financial flows were not of sufficient magnitude to explain transmission across borders on their own. Now, with a crisis that originated in the financial sector, the connections between economies are of sufficient magnitude to merit deeper exploration. Globalization occurred rapidly throughout the past 30 years, and the growth of U.S. financial flows reflects this. Specifically, in the lead up to the financial crisis in 2008, we observed the steepest increase in financial flows, going from roughly \$5 trillion in 2000 to a little under \$20 trillion in 2007.

Our theory of contagion is contingent not only upon financial flows, but upon the type of financial flow in question. We argue that, as shown in Mendoza et al. (2009), the richness of financial markets determines the composition of flows and that countries like the U.S. typically purchase riskier capital assets abroad rather than safe debt instruments. Contagion occurs when these developed countries purchase significantly fewer productive capital assets. Figure 2.2 shows productive capital outflows (notationally, for the remainder of the paper any references to capital flows will refer to non-debt flows, while debt flows will refer to debt alone) from the United States from 1980 to 2011, calculated as the log difference in the stock of the sum of foreign direct investment and portfolio equity investment. The blue dotted line graphs these changes, while the red dashed line provides the average capital flow growth

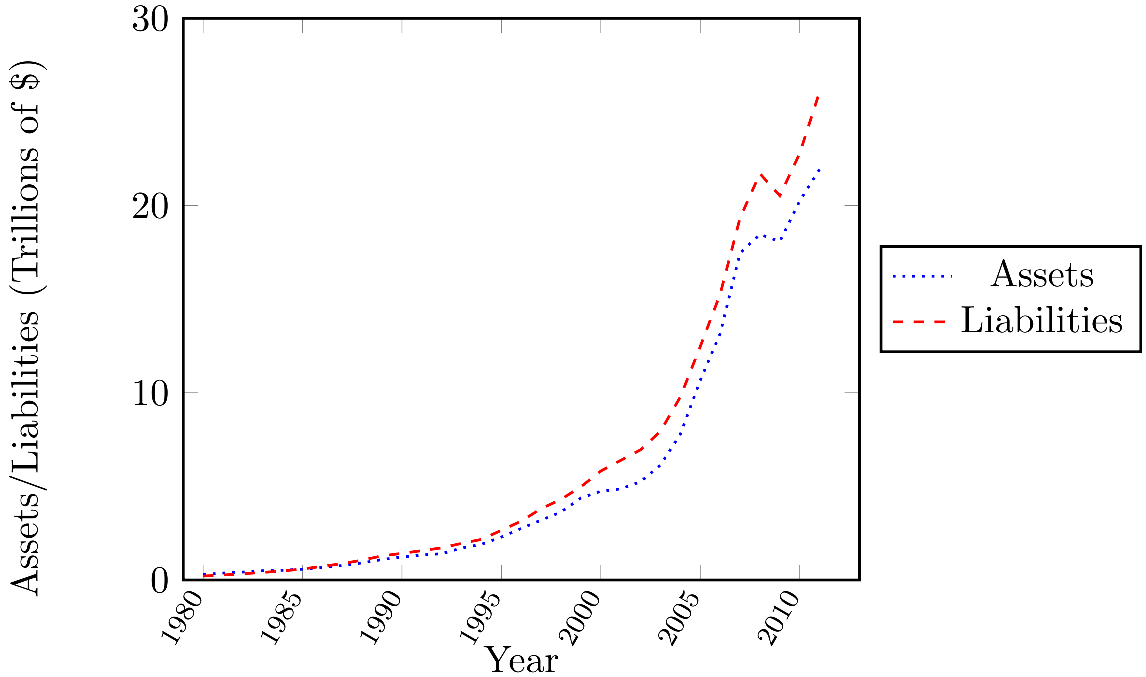


Figure 2.1: Total U.S. Assets And Liabilities (Constant 2011 $)

rate prior to the crisis. What is immediately noticeable is the decline in capital outflows during the crisis is a significant outlier, roughly 30 percentage points below the average capital outflow. As observed in Figure 2.1, we observe a real decline in asset holdings, and a sharp break from the trend growth.

Another key question is whether this decline is driven by the size of the U.S. economy or the depth of the U.S. financial market. Figures 2.3 and 2.4 attempt to illustrate that it is the quality of financial markets that driven this change rather than the size of the economy. Figure 2.3 shows the log differences of the capital holdings for the United Kingdom. Apart from the United States, the United Kingdom has among the deepest financial markets in the world, but they are not as large an economy. The behavior of capital outflows for the United Kingdom is remarkably similar to that of the United States. Both have average capital outflow growth of slightly over 10% in the lead up to the crisis and feature a stunning collapse during the crisis. Figure 2.4 shows the log differences of the capital holdings of Japan. Japan is among the largest economies during this time frame, both in terms of production and in terms of exposure to foreign trade, but does not have the same financial development as the United States or the United Kingdom. Here the break in capital flow growth is earlier, around roughly 1992. During the financial crisis the decline is nowhere near as pronounced as that for the United States. The United States is unique in its combination of the size of its economy and the depth of its markets, but, from these

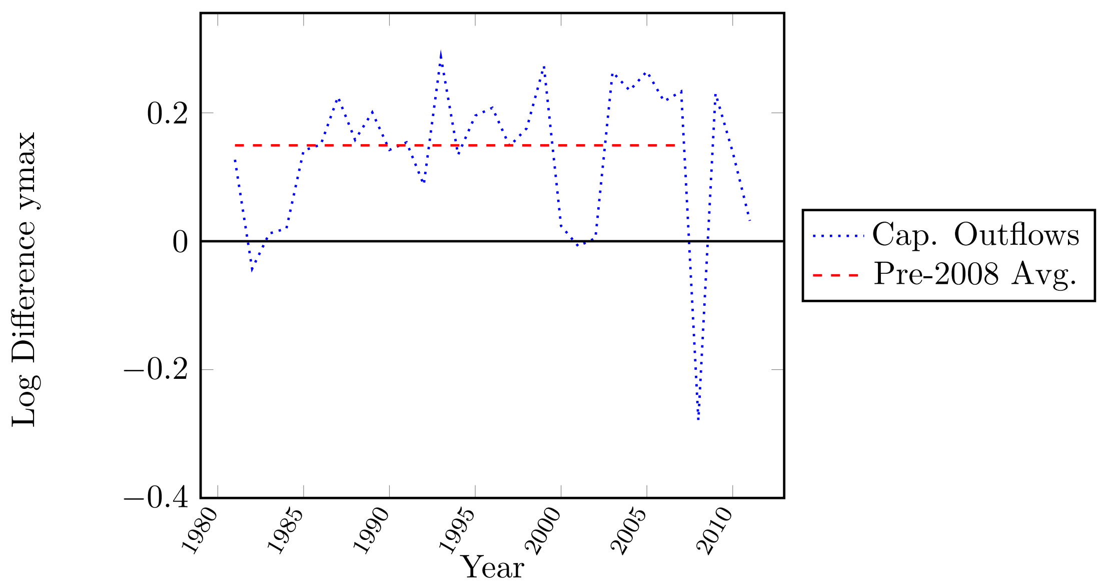


Figure 2.2: Total U.S. Capital Flows (Log Diff. of Stock)

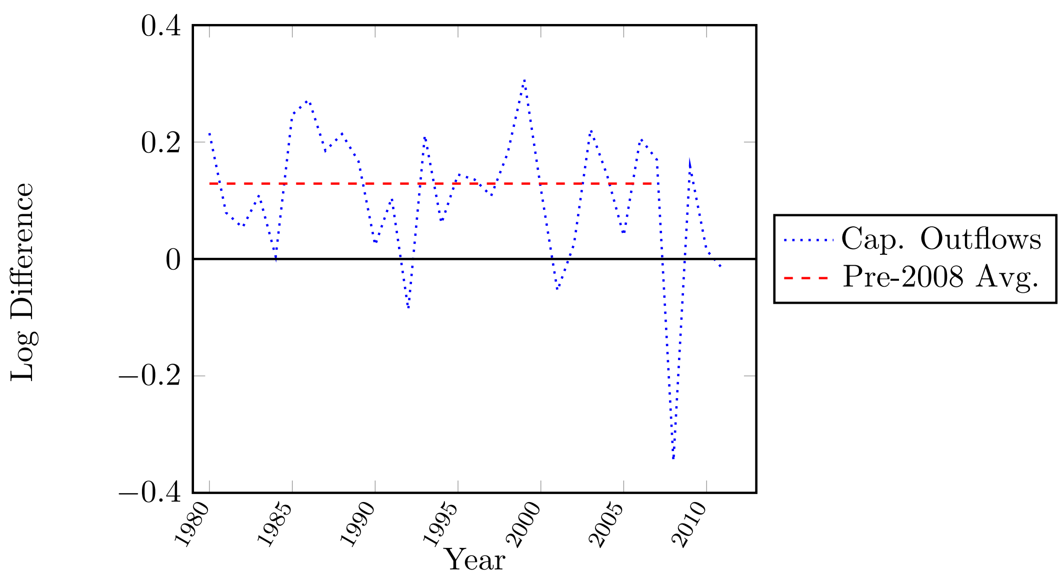


Figure 2.3: Total U.K. Capital Flows (Log Diff. of Stock)

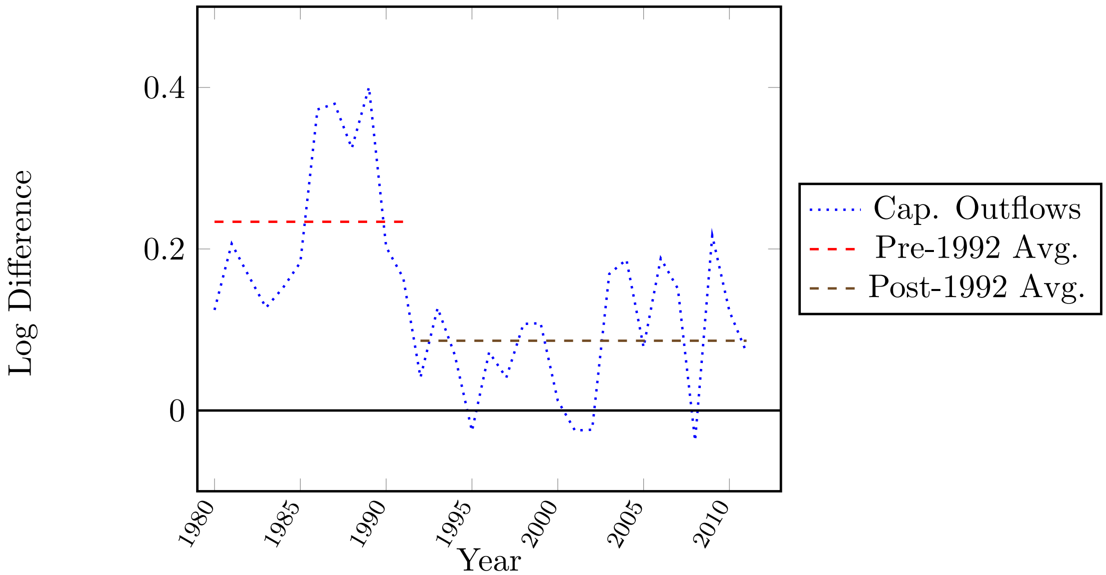


Figure 2.4: Total Japan Capital Flows (Log Diff. of Stock)

charts, we believe that the driving force behind the retrenchment of capital flows is the structure of its financial markets.

The following charts attempt to document the importance of United States capital outflows to the rest of the world. The data for these charts comes from the TIC database, which is sparser in coverage in terms of countries than the Lane and Milesi-Ferretti database. For these charts, we look at the change in U.S. capital inflows into five countries: Germany, Canada, Mexico, Korea, and Japan. Figure 2.5 shows the log difference in U.S. capital holdings of foreign assets in Germany, Canada, and Mexico. Figure 2.6 shows the GDP growth rate in Germany, Canada, and Mexico. First, notice that both series are highly volatile, and that the volatility of capital flows is an order of magnitude higher than the volatility of GDP growth. In general, for these countries, it is difficult to see a relation between the financial flow retrenchment and the transmission of the global recession. All three countries do feature a decline around the recession, and it appears that the larger the change in U.S. outflows the larger the change in output. The result clearly holds best for Mexico, a less-developed economy than Germany or Canada.

It is easier to see the relationship between financial flows and output for Korea and Japan. Again, Figure 2.7 shows the log difference in the stock of U.S. capital outflows to Korea and Japan, while Figure 2.8 shows GDP growth for Korea and Japan. Here it is clear that during both of the most recent recessions for Korea, the East Asian Tiger Crisis in 1998 and the financial crisis in 2008, U.S. capital inflows decreased by a significantly larger amount than the standard fluctuations in flows and the GDP growth rate was significantly

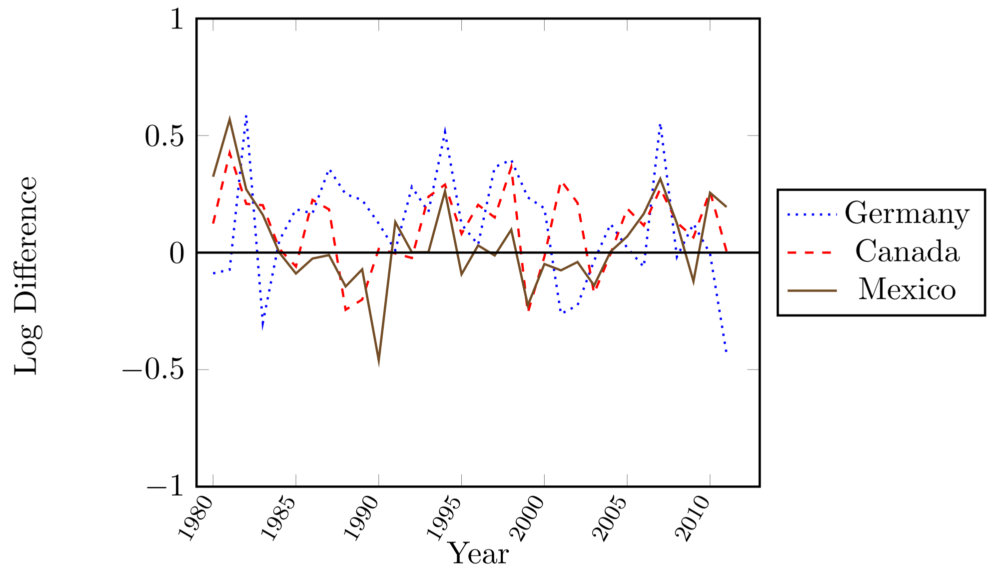


Figure 2.5: U.S. Capital Inflows (Log Diff. of Stock)

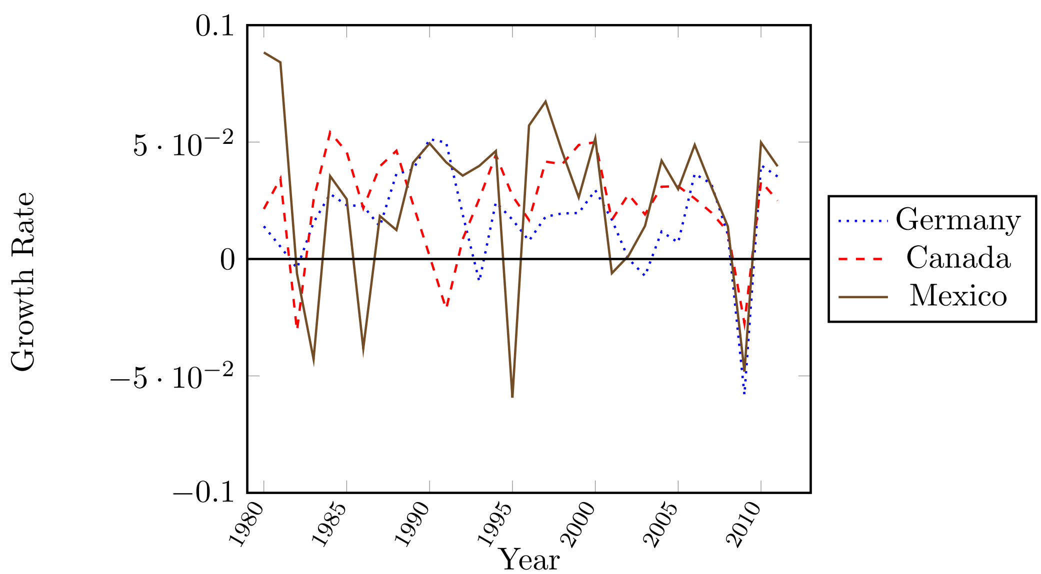


Figure 2.6: GDP Growth

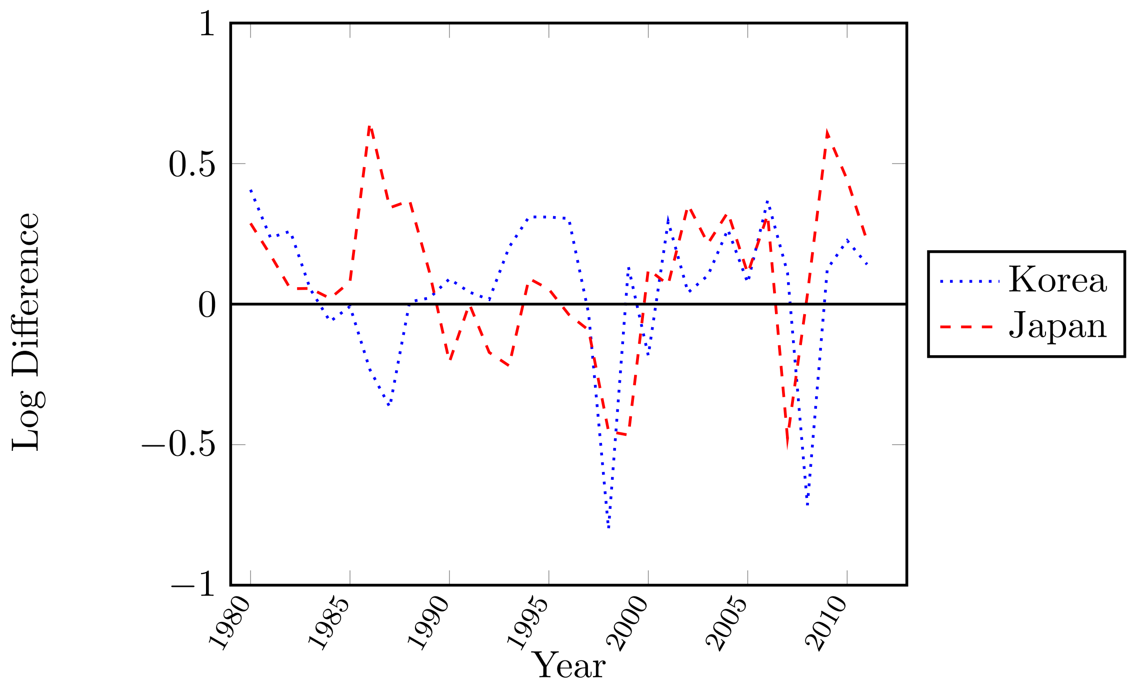


Figure 2.7: U.S. Capital Inflows (Log Diff. of Stock)

lower than the standard fluctuations of GDP growth. During the 2008 financial crisis, Japan also faces the largest decline in financial flows it has seen over the course of the available data, and has the largest decline in output seen in the time frame as well.

To briefly summarize the data to this point, U.S. capital outflows declined sharply in the wake of the financial crisis, and, from a cursory glimpse at the data for Canada, Germany, Mexico, Japan, and Korea, it is difficult to precisely tease out the correlation between U.S. capital flows and output growth in other countries. The following charts try to establish the importance of both the U.S. capital flows to output in other countries, as well as the importance of capital flows generally to the financing of capital stocks the overall relation between financial flows and output. The crisis originated in the United States, but affected all of the most sophisticated financial markets, so looking at the general importance of financial flows is instructive as well. The following two tables, Figures 2.9 and 2.10 show regression results for these two cases.

Figure 2.9 shows the importance of U.S. capital flows to GDP growth abroad for a sample of 42 countries from 1980 to 2011. The regression features many control variables including fixed effects, non-U.S. capital inflows and its lag, the lag of U.S. capital inflows, and other demographic and institutional quality controls. The key result is straightforward: U.S. capital flows have a positive correlation with output growth. This means that the more U.S. capital flows collapsed in the wake of the crisis, the more output was affected. While

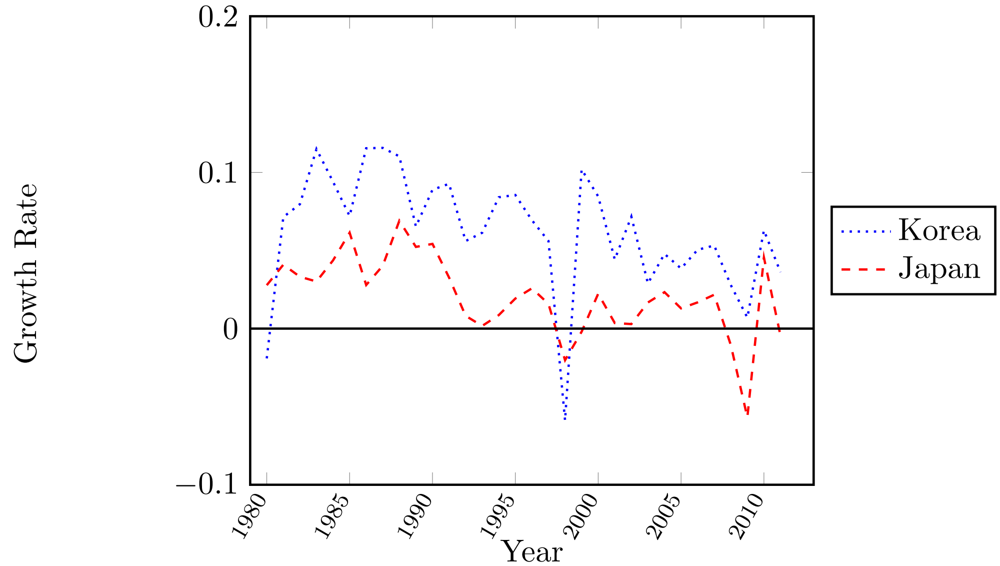


Figure 2.8: GDP Growth

VARIABLES	(1) GDP Growth
U.S. Capital Inflow Growth	0.00262*
	(0.00148)
Controls?	Yes
R-squared	0.323

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Figure 2.9: Regression Results: U.S. Capital Inflows Effect on Output

it would be premature to claim causality for this result, it is important to note that the magnitude of the coefficient is economically significant. From the previous graphs, we can see that U.S. capital inflows fell by almost 50% for Korea and Japan. This coefficient would imply a roughly 10% decline in output, which is double the observed decline. While not perfect, this result indicates that U.S. capital inflows are appropriately correlated and have the potential to be of a sizable magnitude in real economic terms.

Figure 2.10 shows regression results on the generic importance of capital flows to financing a country's capital stock as well as the relationship between capital flows and output growth. This table relies on the Lane and Milesi-Ferretti data set of 113 countries. Again, many control variables are used including fixed effects, lagged growth rates of the listed variables, and other demographic and institutional quality control variables. In the first column, we see the effect of capital inflows on the growth of the capital stock. Prior to the crisis, increases in capital inflows led to increases in the capital stock. After the crisis, the relationship flips. We interpret this as indicating that capital inflows serve as a substitute to private investment. In good times, foreign capital flows in and provides the needed investment, while in bad times, domestic investment is used. Note also that the coefficients on debt flows and labor force have the expected signs. Foreign debt holdings fell during the crisis so savings shifted from debt to financing the domestic capital stock. The coefficient on labor force growth indicates that labor is a substitute.

The second regression result shows the impact of capital flows on the growth rate of GDP. Prior to the crisis, capital inflows are strongly positively correlated with output growth. After the crisis, the relationship preserves the direction but becomes diminished in magnitude. Recall that capital flows dropped by an order of magnitude during the crisis, so the dampened overall coefficient does not imply a smaller overall effect. The key result, however, is the overall positive coefficient. On average, the larger the decline in capital inflows, the deeper the recession was during the crisis.

These two regression results indicate that financial flows are important both in general, and specifically, U.S. outflows are significant. The impact of the recession was asymmetric in that countries that were subject to sharper declines in inflows also faced deeper transmitted recessions. The initial charts show that the major driver of the decline in financial flows originated in more financially developed economies, rather than simply big economies with large trade exposure. From these facts the next section will introduce a model that can generate asymmetric transmission based upon the work of Mendoza et al. (2009).

2.3 Model

This section introduces and characterizes a model based upon the household problem from Mendoza et al. (2009). Our model differs in a few key dimensions: We allow for capital accumulation, we have different masses of individuals in each country, and, rather than having common stochastic processes across countries, we parameterize the income and productivity processes to match observed moments in the data. For the remainder of this section, we will introduce the environment of the model, and then detail the household problem and market clearing conditions. After defining the equilibrium object, this section concludes with a brief characterization of the model that provides insight into the main mechanism driving our results. For detailed information on the characterization as well as a definition

	(1) Cap. Stock Growth	(2) GDP Growth
Cap. Inf. Growth	0.0195	0.0176**
	(0.0148)	(0.00687)
Cap. Inf. Interaction	-0.0429	-0.0166
	(0.0306)	(0.0104)
Debt Inf. Growth	0.0112	0.0289***
	(0.0204)	(0.00749)
Debt Inf. Interaction	0.0361	-0.0474***
	(0.0493)	(0.0158)
Labor Force Growth	-0.183	0.128**
	(0.225)	(0.0503)
Cap. Stock Growth		-0.00540
		(0.00972)
Constant	0.191	0.133***
	(0.130)	(0.0449)
Controls?	Y	Y
Observations	1,632	1,632
R-squared	0.019	0.079
Number of Countries	113	113

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Figure 2.10: Regression Results: Capital Inflows Impact on Capital Formation and Output

of the equivalent economy with the bond market constraint as opposed to the transformed idiosyncratic shocks presented below, please see the appendices for this chapter.

2.3.1 Environment

To analyze the importance of financial development on the transmission of crises, we use a straightforward model that has both global financial markets and portfolio choice. For purposes of exposition, assume the world economy has two countries, $i \in \{1, 2\}$, though this model can easily be generalized to N countries. Both countries have masses of infinitely-lived agents. In our model, country 1 will represent the United States and other developed nations, while country 2 is the rest of the world. As such the mass of agents will not be equal, with country 2 the larger of the two economies. It is assumed agents maximize expected lifetime utility $E\sum_{t=0}^{\infty}\beta^t U(c_t)$, where β is the intertemporal discount factor. It is assumed that the utility function is strictly increasing and strictly concave.

In each country, households face a portfolio choice problem. At the beginning of each period, households observe their idiosyncratic wage and productive capital shocks. This determines their wealth for the period, which they proceed to allocate across consumption, c, and two types of assets: one period, state-uncontingent bonds, b, and capital, k. Capital holdings are used in an idiosyncratic production function of the form zk^{ν} after a one period delay, where z is the idiosyncratic productivity shock. It is assumed that the production function is of decreasing returns to scale, meaning that $\nu < 1$.

Individuals face two types of idiosyncratic risk, exogenous income risk, y, and an idiosyncratic investment shock, z. Both income risk and the productivity shock are Markov processes. As all shocks are idiosyncratic, and there are no aggregate shocks, there is no aggregate uncertainty. As in Mendoza et al. (2009), the presence of both shocks allows for analyzing a portfolio choice between riskless and risky assets. The endowment shock is unavoidable and can only be insured up to the limit set by the sophistication of the financial markets in that country. On the other hand, it is possible to avoid the productivity shock completely by simply not purchasing any capital.

The key feature of our model is the financial environment in each country that will be fundamental to generating the capital flows and transmission we observe in the data. First, in both countries, households face a limited liability constraint, meaning that net assets must be greater than or equal to 0, which limits borrowing. Second, the quality of financial markets affects the idiosyncratic shocks households face. The variable $\phi \in [0, 1]$ denotes the quality of financial markets. When $\phi = 0$, the household faces the full force of its idiosyncratic shocks. When $\phi > 0$, the household gets a weighted average of the shocks it faces and the conditional expectation of the shock given last period's realization. This lowers variance while preserving expected return, so it captures the idea of efficient diversification.

To summarize, households face a portfolio choice in two countries that are defined by their financial completeness which affects the idiosyncratic shocks they face, and thus the composition of their portfolios. The timing of the model is as follows:

1. Households enter the period holding capital and bonds. Shocks are then observed, which determines current net asset holdings.

2. At the end of the period, households choose consumption (c) and their portfolio for tomorrow(k' and b') given their asset holdings.

The next subsection formalizes the household problem and how financial market depth manifests itself in this environment.

2.3.2 Household Problem

Let the idiosyncratic state be given by $s \equiv (y, z)$, where y is the income endowment the household receives this period, and z is a vector of productivity shocks with the same number of entries as countries in the economy. Specifically, an individual gets an idiosyncratic draw for portfolio productivity in each country. Let $g(s, s')$ be the associated conditional probability distribution. Define a to be assets available at the end of the period, then the household budget constraint is given by Equation 2.1.

$$a = c + k'_1 + k'_2 + b' \tag{2.1}$$

Assets are used to purchase today's consumption, as well as domestic capital, k'_1, foreign capital, k'_2, and bonds, b', all of which are used in the next period. As mentioned above, asset holdings are determined at the beginning of each period, when the idiosyncratic state, s, is realized. Assets for the period are described by the following law of motion:

$$a(s) = y + \tilde{z}_1 k_1^\alpha + \tilde{z}_2 k_2^\alpha + k_1 + k_2 + (1 + r)b \tag{2.2}$$

Where

$$\tilde{z} = (1 - \phi)z + \phi E\left[z|z_{-1}\right] \tag{2.3}$$

Assets are simply the realization of the endowment shock plus both the production and resale value of capital in each country, and the returns on any bonds that come due or the repayment of any debts that are owed. To clarify further, $\tilde{z}$ is the effective realization of the productivity shock. These realizations are determined by the level of financial development in the economy, ϕ_i. As is shown in the appendix, this is equivalent to an economy with state contingent assets and a limited enforcement constraint of the form: $a(s_j) - a(s_1) \geq (1-\phi)\left[(w_j + z_j k^{\nu}) - (w_1 + z_1 k^{\nu})\right]$, where the subscripts are elements of the finite support of the shocks with 1 corresponding to the worst realization of these shocks. This equation simply says that bonds can only make up for part of the difference from your current asset state and the worst asset state. Households have an incentive to claim to be in the worst asset state when they are not, but there is a cost ϕ to hiding assets. This constraint limits the ability of bonds to span the shocks. Our set-up with an uncontingent bond and transformed shocks has an equally easy to interpret meaning. The better the quality of the financial market, the more efficiently households are able to diversify and insure against the idiosyncratic shocks they face. Specifically, households receive a weighted average of the raw shock and the conditional expectation of the shock given last period's shock. This reduces variance, which, given the form of the households' utility function, makes households more willing to hold risky assets as ϕ increases.

The final component of the household problem is a limited liability constraint. Specifically, households must be solvent, regardless of the realization of the idiosyncratic state:

$$a(s_j) \geq 0 \tag{2.4}$$

This acts as a borrowing constraint, as borrowing in the risk free asset beyond the worst possible endowment realization must be offset by a corresponding position in capital. Having now explained the problem the household faces, we are able to state the full optimization problem, write our market clearing conditions, and define our equilibrium. The value function of an individual in country i is given by:

$$\begin{aligned}
V^i(a, s'; \phi^i) = & \max_{c, k_1', k_2', b'} \left\{ u(c) + \beta E \left[V^i(a', s'; \phi^i) \right] \right\} \\
& \text{subject to} \\
a &= c + k_1' + k_2' + b' \\
a(s) &= \tilde{y} + \tilde{z}_1 k_1^{\alpha} + \tilde{z}_2 k_2^{\alpha} + k_1 + k_2 + (1+r)b \\
\tilde{z} &= (1 - \phi^i) z + \phi^i E \left[z | z_{-1} \right] \\
a(s_j) &\geq 0
\end{aligned} \tag{2.5}$$

2.3.3 Market Clearing

There are three markets that need to clear: the goods market, the capital market, and the bond market. Let $\Omega^i(s, k, b)$ is the joint distribution over shocks, capital, and bond holdings in country i.

The goods market clearing condition is given by:

$$\sum_i \int_{s,k,b} c\Omega^i(s,k,b) + \sum_i \int_{s,k,b} k'\Omega^i(s,k,b) = \sum_i \int y_i \Omega^i(s,k,b) + \sum i \int_{s,k,b} z_i k^{\nu} \Omega^i(s,k,b) \tag{2.6}$$

Total consumption and capital investment must equal the total production and sum of endowments for the world economy. The capital market clearing condition is:

$$\sum_i \int_{s,k,b} k^i(s,a)\Omega^i(s,k,b) = \sum_i K_i \tag{2.7}$$

The total capital in the world economy must equal the total capital held in the world economy. The bond market clearing condition is:

$$\sum_i \int_{s,k,b,s'} b^i(s,a,s')\Omega^i(s,k,b)g(s,s') = 0 \tag{2.8}$$

Net bond holdings in the world economy must equal zero. Now that we have market clearing, we are able to define our equilibrium.

2.3.4 Equilibrium Definition

Given the financial development, ϕ^i, of all countries, a general equilibrium with capital mobility is defined by sequences of:

1. agent's policies for consumption, capital, and bonds $\left\{c_t^i(s,a), k_t^i(s,a), b_t^i(s,a,s')\right\}_{t=0}^{\infty}$,
2. value functions, $\left\{V_t^i(s,a)\right\}_{t=0}^{\infty}$
3. prices, $\{\tau_t, r_t\}_{t=0}^{\infty}$,
4. and distributions, $\left\{M^i(s,k,b)\right\}_{t=0}^{\infty}$

Such that:

1. the policy rules solve the household problem (2.5) and $\left\{V_t^i(s,a)\right\}_{t=0}^{\infty}$ are the associated value functions,
2. and markets clear, meaning (2.6), (2.7), and (2.8) hold.

In the following section, we characterize the solution to the household problem

2.3.5 Characterization

While it is not possible to determine an analytic solution for this problem, the characterization provides insight into what the solution will look like. Specifically, showing the Euler equations for each country in a two country model provides insight into the main mechanism of the model. For the full algebraic derivation of the characterization, please see the appendix to the chapter. In this two country example it will be assumed that $\phi^1 > \phi^2$. The Euler equation for capital in country 1 is:

$$\beta E\left[u'(c')(\alpha\tilde{z'}_1 {k'}_1^{\alpha-1}+1)\right] + E\left[\mu(s')\alpha(\tilde{z'}_1 {k'}_1^{\alpha-1}+1)\right] = u'(c) \tag{2.9}$$

The Euler equation for capital in country 2 is:

$$\beta E\left[u'(c')(\alpha\tilde{z'}_2 {k'}_2^{\alpha-1}+1)\right] + E\left[\mu(s')\alpha(\tilde{z'}_2 {k'}_2^{\alpha-1}+1)\right] = u'(c) \tag{2.10}$$

The Euler equation for bonds is:

$$\beta E\left[u'(c')(1+R(A_1,A_2))\right] + E\left[\mu(s')(1+R(A_1,A_2)\right] = u'(c) \tag{2.11}$$

Before attacking the case where there is financial mobility, it is appropriate to examine the case of financial autarky. Define the gross rate of return on capital in country 1 to be $R_1(z,k) \equiv (\alpha \tilde{z'}_1 k_1'^{\alpha-1} + 1)$, and the gross rate of return in country 2 to be $R_2(z,k) = \alpha \tilde{z'}_2 k_2'^{\alpha-1} + 1$. If ϕ^1 is sufficiently high to ensure complete markets (in the case of iid shocks, this is $\phi^1 = 1$, while, with persistence, it must be greater than 1), then the expectations disappear and we are left with a certain outcome. This equates $R_1(z,k) = 1 + r$, and we know that $\beta(1+r) = 1$. Suppose now that country 2 has the polar opposite case of truly incomplete markets that cannot span any uncertainty, $\phi^2 = 0$. Then it must be that $\beta(1+r) < 1$, and $E\left[R_2(z,k)\right] = (1+r)$.

What happens then when we move to financial integration? The above tells us that the risk-free rate of return is lower in country 2 than country 1, so when moving to financial integration the risk free rate will move to an intermediate value between the two autarky rates to clear the bond market. Country 2 demands more risk-free bonds, while country 1 wants to borrow to finance more capital investment. Thus, the ordering of ϕ across countries determines the composition of flows, while the size of the gap between them determines the magnitude of the flows. These are the key insights to the mechanism in our model. If we interpret a financial crisis as a change in the ability of financial markets to span shocks, meaning a decrease in ϕ, we can replicate the observed pattern in flows. When the ϕ in the more developed country decreases, presuming it does not fall so much as to be less developed than the other country, the composition of the flows will stay the same, but the magnitude of the flows will decrease. This means the trade in bonds will collapse, and more importantly for transmission, financing of capital in the less developed country will decrease. With all this in mind, the next section will detail the computational algorithm and parameterization of the model.

2.4 Computational Algorithm and Parameterization

This section details the experiment we run and the computational algorithm and the parameterization of our model. The experiment we run is straight forward. We assume the economy was in steady state prior to the financial crisis. The financial crisis manifests itself as a sharp decrease in country 1's ϕ. We then solve for the new steady state at this lower ϕ, as well as for the transition path between the two steady states. The following subsection explains the computational algorithm for both solving the steady states and the transitions.

2.4.1 Computational Algorithm

First, we need to obtain the distribution and market clearing interest rate associated with each steady-state equilibrium. To solve for a given steady state:

1. Pick an interest rate.
2. Find policy functions for $k1$, $k2$, and b for each country.
3. Simulate economy to find net borrowing and capital holdings.
4. Check market clearing conditions (e.g. Net borrowing equal to zero, target aggregate capital, goods market clearing).
5. Repeat from step 1 until within error tolerance

Once we solve for the steady states, we obtain the associated distributions, call them Ω_{bef} and Ω_{aft}, and interest rates, r_{bef} and r_{aft} with subscript bef corresponding to the steady state *before* the crisis and subscript aft to the steady state *after* the crisis. Given these distributions and interest rates, we can solve for the transition dynamics using the following algorithm:

1. Guess r_i $i \in n$, where n is the number of transition periods
2. Using the sequence of $\{r_i\}$ and the new steady-state's value function, V_n, solve backwards for $V_{n-1}, \ldots, V_0$
3. Using the sequence of value functions, $\{V_i\}$, find the sequence of $r_0, r_1, \ldots, r_{n-1}$ which clear the bond market
4. If $\Delta V_i < \epsilon_i \; \forall i \in n$, end, otherwise go to (1).

Now that the algorithm has been specified, the next subsection provides the parameterization of the model.

2.4.2 Parameterization

Table 3.1 provides the parameter values we use in our numerical experiments. When parameterizing the model, we use standard values for as many parameters as possible. Specifically, we set the capital share parameter in our production function, $\alpha = 0.33$, and use a CRRA parameter of $\sigma = 2$ in our utility function. For our discount rate, we use a β of 0.95, which, in the case of complete markets, implies an interest rate of around 5%.

Parameter	Value
Discount Rate, β	0.95
Capital Share, α	0.33
CRRA Parameter, σ	2
Country 1 Prod. Shock Mean, z_1	0.08
Country 1 Prod. Shock SD, σ_1	0.20
Country 2 Prod. Shock Mean, z_2	0.10
Country 2 Prod. Shock SD, σ_2	0.25

Table 2.1: Parameter values

To discipline the stochastic processes governing the returns to capital, we look at stock market data and investment prospectuses from companies such as Morgan Stanley Capital International (MSCI) and Goldman Sachs. In general, when looking at the return on the U.S. stock market, we find that a mean of around 7-8% with a roughly 20% standard deviation is consistent across the various sources. For foreign returns, MSCI indicates a mean return of roughly 10-11% with a standard deviation between 22-25% for foreign capital.

For the income process, we first notice that the U.S. accounts for roughly half of the world's gross domestic product, while the per capita income is roughly 3 times the average from the rest of the world. This means that we set the size of the rest of the world to be 3 times that of the U.S. and make the income process have a mean that is one third of the U.S.'s mean.

Finally, for our numerical exercise, we need to pin down the quality of financial markets, ϕ, in each country. For simplicity, we will think of country 2 as completely financially-undeveloped, meaning that $\phi_2 = 0$. Regarding the U.S. financial market value, we look at Mendoza et al. (2009) and see that obtaining a precise value for this ϕ is difficult. For illustrative purposes, we will start with a ϕ_1 of 0.8 prior to the crisis. After the crisis, $\phi_1' = 0.6$, meaning that the crisis reduced market efficiency by 25% of its initial value. Now that the model has been parameterized, we present the solutions to both the steady states as well as the transition path between them.

2.5 Results

This section provides the results of our numerical exercise. In this exercise, we first calculate two steady states, and then solve for the transition dynamics between them. Generally speaking, we observe patterns consistent with the data documented above: the gross magnitude of bond flows sharply declines, and, in the immediate aftermath of the crisis, there is

	Country 1	Country 2
K_1	4.61	3.94
K_2	7.92	6.86
b_i	-10.29	10.61
A_i	6.15	18.55
ϕ_i	0.8	0.0

Table 2.2: Pre-Crisis Steady State

	Country 1	Country 2
K_1	6.96	4.71
K_2	11.60	7.93
b_i	-3.63	3.70
A_i	20.33	18.01
ϕ_i	0.6	0.0

Table 2.3: Post-Crisis Steady State

a correlation between the decline in foreign capital holdings of the well-developed country and the total asset holdings of the less-developed country, indicating the crisis has been transmitted.

2.5.1 Steady State

Tables 2.2 and 2.3 provide the steady state results for our model and calibration pre- and post-crisis. From Table 2.2, we notice that prior to the crisis most capital is held by country 1, and that these holdings are financed by large amounts of borrowing. The average overall asset holdings for Country 2 is roughly three times that of Country 1, as was targeted. The equilibrium interest rate is roughly 7%.

Table 2.3 shows that after the crisis occurs Country 1 drastically changes its portfolio composition. As there is less insurance available, we observe a strengthened precautionary savings motive that leads to significant increases in productive capital holdings and retrenchment in bonds for both countries. Again, the vast majority of capital is held by Country 1, though Country 2 has also significantly increased their holdings as well. As with the data, the gross magnitude of bond flows drops significantly after the crisis, and the equilibrium interest rate is now roughly 5%. This lower interest rate is a result of the increased demand for bonds, which raises their price. Unlike the data, asset holdings increases significantly, but this is unsurprising in the new steady state. The lack of insurance means that asset accumulation to insure against shocks. The next subsection will look at the transition dynamics in the wake of the crisis to see if the crisis was transmitted.

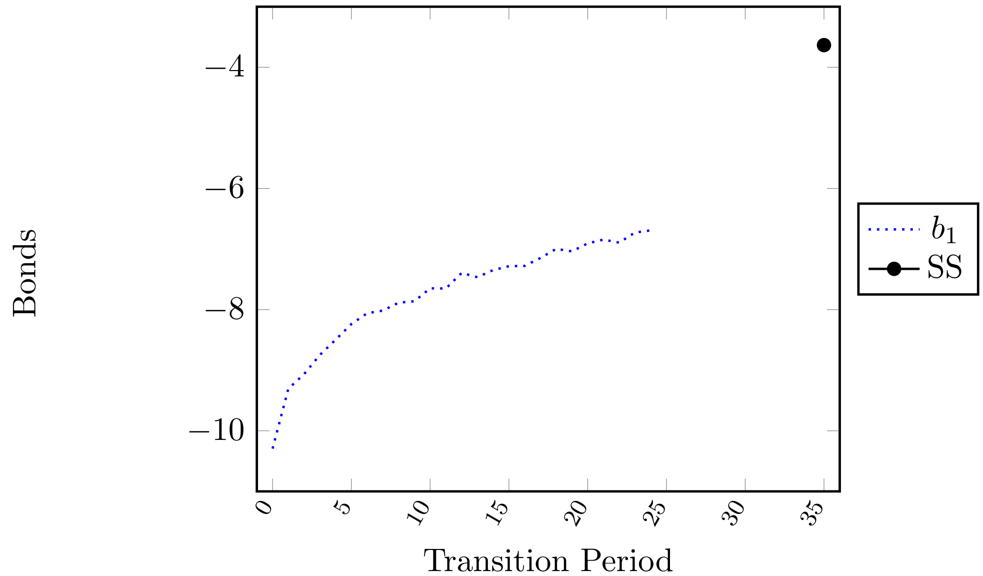


Figure 2.11: Bond Holdings (Total)

2.5.2 Transition Path

Figure 2.11 shows the transition path of bond holdings in the wake of the crisis. The blue line is the transition path of bond holdings in country 1. As net bonds in the global economy must be zero, the transition path of bond holdings for the rest of the world is simply the mirror image of this plot. The black dot is the new steady state value for bond holdings. The transition to this value is smooth, but takes time to get there. What we notice is an immediate decline of roughly 25% of the volume of bond flows in the global economy, which is of the same order of magnitude as observed in the data.

Figure 2.12 shows the transition path of Country 1's capital holdings. This shows that, initially, capital holdings decline prior to slowly but steadily recovering. This is a result of less insurance being available. Households want to initially reduce capital holdings until they can sufficiently increase their asset holdings because of an increased precautionary savings motive. This decline in overall financial flows clearly leaves Country 2 worse off, as the next image shows.

Figure 2.13 shows the transition paths of asset holdings for both countries. Notice that, after the crisis, Country 1 steadily accumulates assets and smoothly transitions to the new steady-state value of around 20 units. Country 2 on the other hand declines in asset holdings for a long while, even though in the long run they also end up with higher asset holdings in the new steady state. This is the transmission of the crisis. Foreign individuals are worse off both initially and for a long duration of the transition.

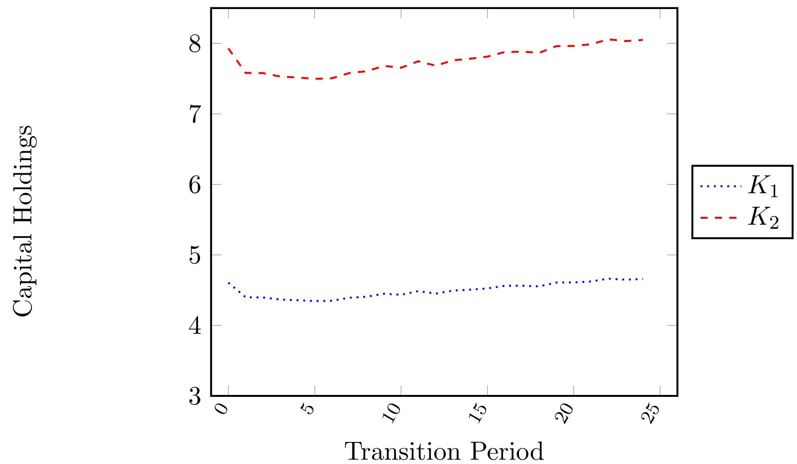


Figure 2.12: Country 1 Capital Holdings

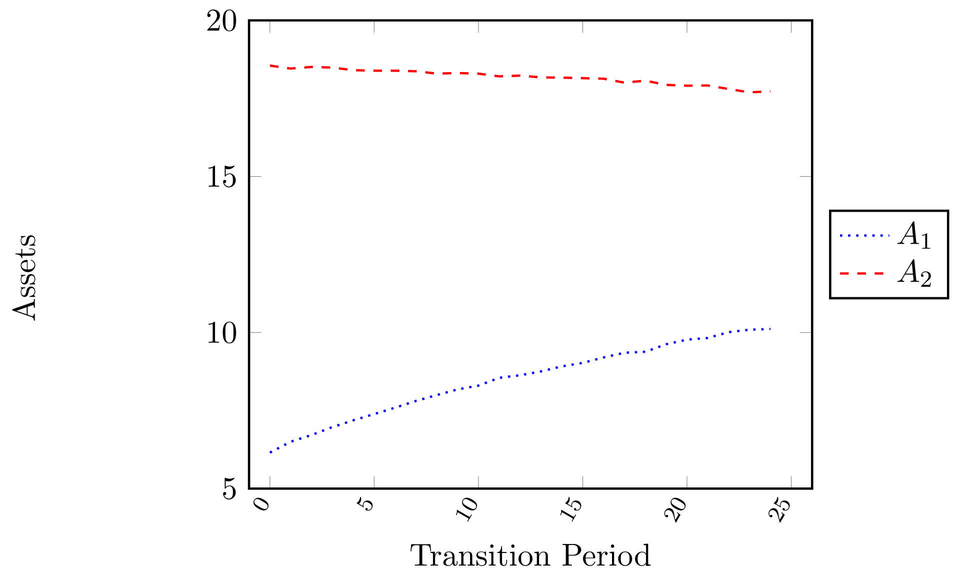


Figure 2.13: Asset Holdings (Total)

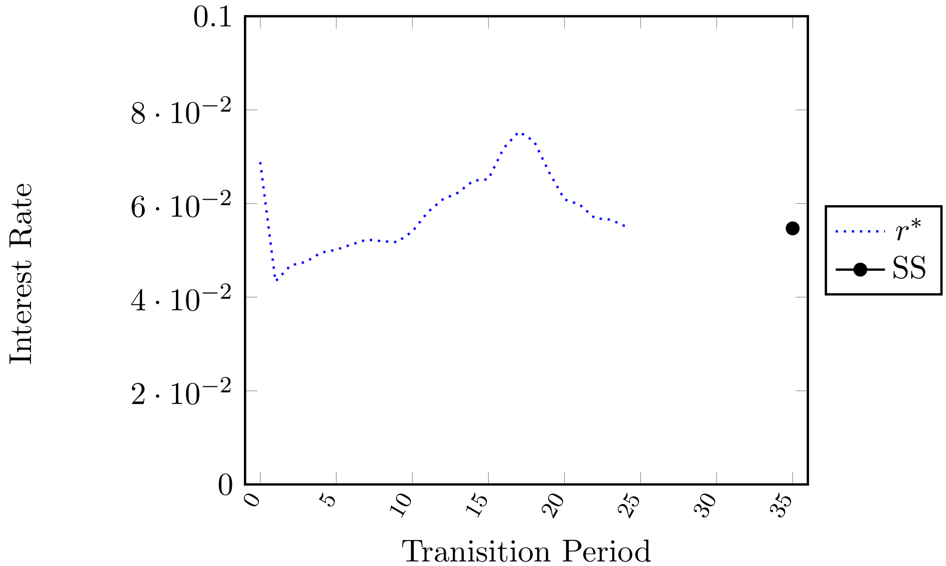


Figure 2.14: Equilibrium Interest Rate

Figure 2.14 shows the transition path of the equilibrium interest rate. This graph indicates that demand for bonds drastically increases following the crisis. The lack of insurance increases the demand for safe assets in Country 1 and correspondingly increases the price of bonds initially. The price falls and interest rate rises as country 1 accumulates assets throughout the transition, overshooting the new steady state value initially, before finally smoothly transitioning to the new steady state value. This occurs because demand for borrowing increases as Country 1 households are able to finance more capital accumulation.

To synthesize the results of these dynamics, the picture painted is broadly that observed in the data. Bond flows collapsed precipitously, while interest rates simultaneously plummeted. Productive capital flows from the financially well-developed economy drop in the wake of the crisis, and we observe a persistent decline in the overall wealth of the lesser-developed country, which is interpreted as the transmission of the crisis.

2.6 Conclusions

Our paper examines the asymmetric transmission of financial crises across borders. We interpret financial crises as disruptions in the ability of financial markets to insure against idiosyncratic shocks. When this occurs in a country like the United States, this changes the portfolio choice and leads to a retrenchment in foreign capital holdings. This decrease in foreign capital transmits the crisis, and, furthermore, hurts foreign countries more the more they rely upon capital inflows.

When looking at data from the Lane and Milesi-Ferretti database as well as the U.S. Treasury International Capital database, we see preliminary evidence that confirms this relationship. Specifically, we observe that financial flows from financially well-developed countries, like the U.S. and the U.K., fell sharply, while flows from large economies, such as Japan, did not behave similarly. We also look into the importance of these capital outflows to the productive ability of those receiving them. For capital flows in general, and U.S. capital flows in particular, there is evidence of an economically significant impact on both output and domestic capital formation.

From this evidence, we propose a model that features financial markets of varying depth based upon Mendoza et al. (2009). In the model, when financial markets no longer intermediate risk as well, countries hold fewer productive assets, leading to less capital in foreign economies, and thus lower production. This transmission is stronger the more reliant upon foreign capital the country suffering from contagion was. We then show in a simple numerical example that the model is able to produce changes of similar magnitude to that observed in the data.

Our paper is, as far as we know, the first to feature asymmetric transmission, meaning that the originating country of the shock is fundamental to the spread of the shock, and that the countries that suffer from transmission do not suffer equally. In fact, it is possible that foreign countries suffer more than the initially affected country. This leads to many possible future research directions. Are more developed economies truly unaffected by crises abroad? Or are they simply not negatively affected because demand for safe assets increases which leads to cheap credit in developed economies? From a technical standpoint, turning the quality of financial markets into a stochastic process would require outside of the box thinking and development of new solution techniques. In the case of financial market quality being subject to aggregate shocks, the interest rate would depend on the distribution of assets, which, as shown in Sager (2015), is not currently solvable.

To conclude, we have presented a model that explains asymmetric international transmission of financial crises. When looking at the data, we find evidence that financial market quality is fundamental to the composition of financial flows, and that capital inflows drop sharply during transmitted crises. Our model shows that, when taking into account financial market depth, we are able to generate transmission of similar magnitude to that in the data.

Chapter 3

An alternative method of solving for endogenous grids

3.1 Introduction

The endogenous grid method (EGM) introduced by Carroll (2006) significantly reduces solution times for a wide variety of dynamic optimization problems with a single continuous control variable and state space. There have been several extensions to the method, but until recently, there has been no generalized solution method for multiple continuous control and state variables. The primary difficulty lies in multi-dimensional interpolation on irregular grids. Several recent papers have addressed this issue, and this paper contributes to this literature. The method described herein uses multi-dimensional spline interpolation with automatically calculated first order derivatives.

Carroll (2006) presented solution is to the problem with a one-dimensional state space with a single, continuous, choice variable. Subsequent solutions have extended the number of choice variables and allowed for additional discrete state variables, but have struggled to provide a general solution for multiple continuous state and choice variables. For example, Barillas and Fernandez-Villaverde (2007) provides a solution for with continuous labor choice, Fella (2014) permits value functions that are non-concave, and the method presented in Iskhakov (2015) permits one continuous variable and one discrete choice variable.

The difficulty with multiple continuous state variables lies in the fact that the gridpoints for interpolation are found by inverting policy functions. The calculated pre-decision state points does not form ordinary mesh, and so standard interpolation technique using a rectilinear grid do not work.[1] Ludwig and Schön (2013) show that Delaunay interpolation can be used to overcome this issue in two dimensions, but suffer from the cost of calculating the triangulation in each period. As the grid size increases, the method slows down, with large grids being slower than tradition methods.

White (2015) presents a new interpolation technique that uses the property that pre- and post- solution points are ordered identically, to skip the need for interpolation construction or function evaluation. The result is a significant increase in the speed and effectiveness of interpolation in multiple dimensions.

This paper proposes a new method of interpolation. Whereas previous solutions attempt to either turn the endogenous grid into a regular form (White (2015) and Druedahl and Jørgensen (2017)) or perform costly interpolation on irregular grids via "visibility walks" (Ludwig and Schön (2013)), this method performs radial basis function (RBF) interpolation using a custom basis based on the utility function.

The choice of a RBF intepolation techniques allows accurate higher dimensional interpolation on irregular grids, as is the standard result when solving endogenous grid problems. An issue with this technique in general implementations is that it can often be quite slow, bypassing the benefit of skipping the maximization step. However, by using a custom basis that closely resembles the final solution, we can significantly reduce the number of points required, reducing the "curse of dimensionality" in higher dimensions.

This benefit is tempered by the cost associated with solving the basis. Therefore, finding the optimal grid size is a non-trivial problem, which potentially limits the benefits associated with this method.

The remainder of this paper is structured as follows: 3.2 provides some technical background on interpolation and RBFs, 3.3 presents the model to be solved, 3.4 presents the algorithm for solving using the endogenous grid method, 3.5 provides results, and 3.6 provides some conclusions.

[1] See White (2015) for more details.

3.2 Technical Background

3.2.1 Interpolation

Given an unknown function, $f(x)$, and a set of discrete points, $S = \{(x_i, f_i(x_i)), 0 \leq i \leq n\}$, our goal is to find a function, ψ such that $\psi(x_i) = f(x_i), \forall (x_i, f(x_i)) \in S$. While there are infinitely many functions that could satisfy this condition, if we limit the functions to polynomials, then for all $m \leq n$, $\psi_m(x)$ is unique.

Newton's Form

Consider the following series of functions:

$$\psi_0(x) = f(x_0)$$

$$\psi_1(x) = \psi_0(x) + a_1(x - x_0)$$

$$\cdots$$

$$\psi_n(x) = \psi_{n-1}(x) + a_n(x - x_0) \cdot \ldots \cdot (x - x_{n-1})$$

If we set

$$a_1 = \frac{f(x_1) - \psi_0(x)}{x_1 - x_0}$$

then obviously $\psi_1(x)$ goes through both x_0 and x_1. Continuing, we can define

$$a_i = \frac{f(x_i) - \psi_{i-1}(x_i)}{\prod_{j=0}^{i-1}(x_i - x_j)}, 1 \leq i \leq n$$

to get a polynomial that goes through all the interpolation points. This series of polynomials is known as the Newton Form. While there may be multiple ways to write this function, the uniqueness of the polynomial guarantees that the values will always be the same.

Divided Differences

We can see that for each i, $a(i)$ is a function of $x_0 \ldots x_i$. If we denote this dependence using the following notation:

$$a_i = f\left[x_0, \ldots, x_i\right]$$

then it can be shown that

$$f\left[x_0,\ldots,x_i\right]=\frac{f\left[x_1,\ldots,x_i\right]-f\left[x_0,\ldots,x_{i-1}\right]}{x_i-x_0}$$

Furthermore, given the uniqueness of the representing polynomial, if we let $z_0,\ldots,z_n$ be a permutation of $x_0,\ldots,x_n$, then it must be true that $f\left[y_0,\ldots,y_n\right]=f\left[x_0,\ldots,x_n\right]$.

Hermite Interpolation

To this point, the goal has simply been to interpolate the function, f. However, it may also be that we want to interpolate the derivatives of the function as well. Interpolation which attempts to accomplish both these tasks is known as Hermite Interpolation. Note that we do not restrict ourselves to the first derivative, but also permit higher order derivatives.

One requirement to obtain a unique interpolant is that for any x_i in which we wish our interpolating polynomial, ψ, to match the j-th derivative of our unknown function, f , is that we must also be given all lower order derivative values at that point. In other words, if we want $\psi^{(j)}(x_i)=f^{(j)}(x_i)$, then we must also be given $\psi^{(k)}(x_i), 0\leq k\leq i-1$. If this condition does not hold, finding a unique interpolate may be impossible.

To obtain the Newton form of the Hermite interpolation polynomial, we first start with divided differences. Note that

$$f'(x)=\lim_{x\to 0}\frac{f(x)-f(x_0)}{x-x_0}=\lim_{x\to 0}f\left[x,x_0\right]$$

We can then extend the definition of divided differences in the following manner.

Definition 3.2.1. Let $x_0\leq x_1\leq\ldots\leq x_n$. Then

$$f\left[x_0,\ldots,x_n\right]=\begin{cases}\frac{f[x_1,\ldots,x_n]-f[x_0,\ldots,x_{n-1}]}{x_n-x_0}, x_n\neq x_0\\ \frac{f^{(n)}(x_0)}{n!}, x_n=x_0\end{cases}$$

The Hermite interpolation problem is as follows. Given a set of ordered points $x_0,\ldots,x_n$, find the polynomial $\psi(x)$ such that for any point $x_j\in[x_0,\ldots,x_n]$, $\psi^{(k)}(x_j)=f^{(k)}(x_j), 0\leq k\leq d_j$, where d_j is the number of derivatives that need to be solved at the point x_j, and can vary with j. If we let $n=\sum_j d_j$, then we can define a set of ordered points

$$\{y_0,\ldots,y_{n-1}\}=x_{0,0},x_{0,m_1},\ldots,x_{0,m_1-1},x_{1,0},\ldots,x_{1,m_2-1},\ldots,x_{k,0},\ldots,x_{k,m_k-1}$$

giving the interpolation polynomial

$$\psi(x) = f\left[y_0\right] + \sum_{j=1}^{n-1} f\left[y_0, y_j\right] \prod_{k=0}^{j-1} (x - y_k)$$

Consider the following example.

Example 3.2.2. Find the polynomial, $\psi(x)$ that satisfies

$$\psi(x_0) = f(x_0)$$

$$\psi(x_1) = f(x_1)$$

$$\psi^{'}(x_1) = f^{'}(x_1)$$

We know from the equation above that

$$\psi(x) = f(x_0) + f\left[x_0, x_1\right](x - x_0) + f\left[x_0, x_1, x_1\right](x - x_0)(x - x_1)$$

The divided differences are given by

$$f\left[x_0, x_1\right] = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$f\left[x_0, x_1, x_1\right] = \frac{f\left[x_1, x_1\right] - f\left[x_1, x_0\right]}{x_1 - x_0} = \frac{f^{'}(x_1) - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_1 - x_0}$$

Therefore, the interpolating polynomial is given by

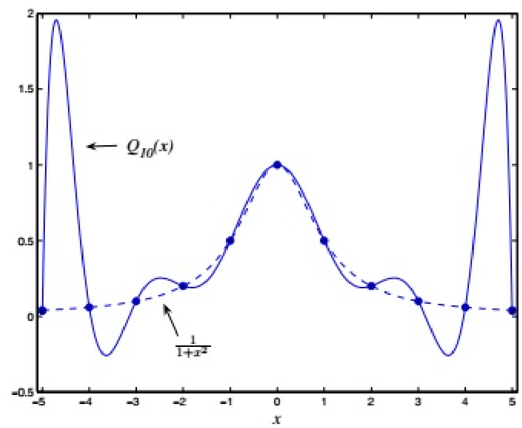
$$\psi(x) = f(x_0) + \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_0) + \frac{f^{'}(x_1) - \frac{f(x_1)-f(x_0)}{x_1-x_0}}{x_1 - x_0}(x - x_0)(x - x_1)$$

Splines

One issue with high order polynomials is that they may have many local extrema. Even when estimating a smooth function with a single extremum, polynomial functions can have significant issues. Figure 3.1 provides an example of this, where a degree 10 polynomial is unable to accurately approximate the given function.

A solution to this problem would be to use a piecewise-continuous polynomial. For example, a piecewise-continuous linear function would simply be the lines connecting each of the points. It would be unlikely for this function to be differentiable at any of the

Figure 3.1: Degree 10 polynomial to interpolate $\frac{1}{1+x^2}$



interpolation points, however, so we may want some concept of "smoothness" at connection points. A spline provides such an attribute.

Definition 3.2.3. A spline of degree n having knots $k_1, \ldots, k_k$ is a function $s(x)$ that satisfies the following two properties:

1. On the interval $[k_{i-1}, k_i), s(x)$ is a polynomial of degree $\leq n$
2. On the interval $[k_0, k_k), s(x)$ has a continuous $(n-1)^{th}$ derivative

So, a degree 0 spline would be the piecewise-constant function (which would not be continuous) and a degree 1 spline would be the piecewise-linear function. While it may be convenient for the interpolation points and the knots to be the same, this isn't necessary.

Cubic Splines. A cubic spline function is a polynomial of degree at most three that connects at the know points. By construction, it also has continuous first and second derivatives at the knot points. Finally, by convention, the knots are chosen to be known interpolation points. This yields the following set of equations

$$s(x_i) = y_i, 0 \leq i \leq n$$

$$s_{i-1}(x_i) = y_i = s_i(x_i), 1 \leq i \leq n-1$$

$$s_{i-1}^{'}(x_i) = s_i^{'}(x_i), 1 \leq i \leq n-1$$

$$s_{i-1}^{''}(x_i) = s_i^{''}(x_i), 1 \leq i \leq n-1$$

Using the notation

$$m_i = x_{i+1} - x_i$$

$$n_i = s^{''}(x_i)$$

and assuming that $n_0 = n_n = 0$ (i.e. the natural spline condition) it can be shown that

$$\begin{bmatrix} 2(m_0+m_1) & m_1 & 0 & \dots & 0 & 0 \\ m_1 & 2(m_1+m_2) & m_3 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & \dots & \dots & \dots & m_{n-2} & 2(m_{n-2}+m_{n-1}) \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{n-1} \end{bmatrix} = \begin{bmatrix} \frac{y_2-y_1}{m_1} - \frac{y_1-y_0}{m_0} \\ \frac{y_3-y_2}{m_2} - \frac{y_2-y_1}{m_1} \\ \vdots \\ \frac{y_n-y_{n-1}}{m_{n-1}} - \frac{y_{n-1}-y_{n-2}}{m_{n-2}} \end{bmatrix}$$

3.2.2 Radial Basis Function Interpolation

While splines work well for one dimension (possibly parameterized) or on regular grids (when in multiple dimensions), they suffer from significant performance penalties when applied to irregular grids. The primary difficulty lies in identifying the point(s) closest to the interpolation location from which to build the spline.

An alternative solution to interpolation on irregular grids is that of using radial basis functions (RBFs). If ψ_m is a basis function, then RBF interpolation is attempting to find ψ such that

$$\psi(x) = \sum_{m=1}^{N} c_m \psi_m(x), \ x \in \mathbb{R}^d$$

The solution to this equation is the solution to the linear equation

$$Ac = y$$

where A, the interpolation matrix, is given by $A_{ij} = \psi_j(x_i)$. For the solution of this equation to both exist and be unique, it must be true that A is non-singular.

Definition 3.2.4. A function $\Gamma : [0, \infty)$ is completely monotone if it satisfies

$$(-1)^k \Gamma^{(k)}(r) \geq 0, r > 0, k = 0, 1, \dots$$

Furthermore, it can be shown that a function is completely monotone and non constant if and only if it is positive definite on $\mathbb{R}^d, \forall d$.

Therefore, as long as the interpolation function, ψ_j is completely monotone, it will generate an interpolation matrix, A, that is positive definite, and therefore non-singular.

3.3 Model

To remain consistent with previous literature, we use the same benchmark model as White (2015)and Ludwig and Schön (2013). Specifically, the model contains an individual who allocates resources between consumption, health, and savings. Individuals obtain utility from consumption, but have a probability of death that is a function of their health. Return on savings is given by an exogenous interest rate, r.

3.3.1 Environment

We present a standard infinitely lived agent model, where households choose between consumption, c, savings, k, and investment in health, η. Individuals obtain utility from consumption, and have standard CRRA preferences given by

$$u(c) = \frac{c^{1-\rho}}{1-\rho}$$

Again, following prior literature, we will assume that $\rho < 1$, ensuring that the value function is strictly positive (and therefore, individuals value extending life). Individuals enter each period with savings (k) and health (h). Income is a function of a stochastic wage draw, w, and health, and is given by

$$I(h) = \begin{cases} wh, w \sim \mathcal{N}(\bar{w}/(1-p), \sigma^2), \text{ w/ prob} = 1-p \\ 0, , \text{ w/ prob} = p \end{cases}$$

There is no borrowing in this model, and so households are subject to the balanced budget constraint

$$k_t + I_t - c_t - \eta_t \geq 0$$

Health production exhibits decreasing returns to scale, and will be given by

$$f(\eta) = (\gamma/\nu)\eta^{\nu}$$

Again, we will assume that $\nu < 1$ so that investment in health always yields a positive return.

Savings have a rate of return, r, and do not depreciate. Health depreciation is stochastic, and given by $\delta \sim U[\bar{\delta} - \sigma_\delta, \bar{\delta} + \sigma_\delta]$. Note that this implies households face both permanent and temporary income risk. Health in the next period is given by

$$h_{t+1} = (h_t + f(\eta_t))(1 - \delta_{t+1})$$

Finally, health affects the probability of survival to the next period, where this probability is given by

$$s(h) = 1 - \frac{\phi}{1+h}$$

3.3.2 Solving

The model can be solved using the traditional method of value-function iteration, where the individual value function is given by

$$\begin{aligned}
& V(k,h) = \max_{c,\eta} u(c) + \beta \int s(h')V(k',h')dF(w,\delta) \\
st \quad & \\
& k - c - \eta \geq 0 \\
& h' = (h + f(\eta))(1-\delta) \\
& k' = (k - c - \eta)(1+r) + wh'
\end{aligned}$$

and where F is a joint distribution on w and δ, and k is the cash-on-hand entering the period.

Given the concavity and non-satiation of the utility function, plus the restriction on risk aversion parameters, we know that the solution is interior. Therefore, the first order conditions are both necessary and sufficient, simplifying the search space for the solution.

This is a standard problem, and so further details of solution algorithms are omitted from this discussion.

3.4 Algorithm for the Endogenous Grid Method

As with the one-dimensional endogenous grid method, a "post-decision" state space is first defined. In this case, the space consists of retained assets $a_t = m_t - c_t - \eta_t$ and health after investment, $H_t = h_t + f(\eta_t)$. This space is the individual's state after making decisions, but prior to the realization of wage and health risk.

The consumer problem can be restated using this new state space, as follows:

$$u'(c_t) = \beta(1+r)\int s(h_{t+1})V^m_{t+1}(m_{t+1}, h_{t+1})dF(w,\delta)$$

$$f'(\eta_t) = \frac{(1+r)\int s(h_{t+1})V^m_{t+1}(m_{t+1}, h_{t+1})dF(\omega,\delta)}{\int(1-\delta)[s'(\cdot)V_{t+1} + s'(\cdot)V_{t+1}(\cdot) + s(\cdot)(wV^m_{t+1}(\cdot) + V^h_{t+1}(\cdot))]dF(\omega,\delta)}$$

st

$$m_{t+1} = a_t(1+r) + wh_{t+1},$$

$$h_{t+1} = H_t(1-\delta)$$

This restatement results in the right-hand sides of both equations being completely determined by the "post-decision" state space. More specifically, given a point in the "post-decision" grid, and iterating through each of the shocks yields h_{t+1} and m_{t+1}. With these values, the choice variables c_t and η_t can be solved. Given the choice variables, and an initial guess for the value functions, we can solve for an updated value of the value function.

The full algorithm is as follows:

1. Build a grid for the "post-decision" state, (a_t, H_t)
2. Guess an initial value for $V_t(a_t, H_t)$
3. Using the initial guess, find V_t^h and V_t^m
4. Iterate through each wage and depreciation shock.
   (a) For the given state, solve for m_{t+1} and h_{t+1}
   (b) Use interpolation to solve for $V_t(m_{t+1}, h_{t+1})$
   (c) Calculate the partial integral for this state
5. Using the value of the integral, solve for c_t and η_t
6. Solve for the original state, m_t and h_t
7. Obtain a new estimate for $V_t(m_t, h_t)$ using the equations for utility and previously calculated values.
8. Use interpolation to estimate $V_{t+1}(a_t, H_t)$
9. return to step 3 until V converges
10. Obtain a new guess

Table 3.1: Parameterization

Parameter	Description	Value
ρ	Coefficient of relative risk aversion	0.5
ν	Curvature of health production function	0.35
γ	Magnitude of health production function	1
ϕ	Maximum death probability	0.5
β	Intertemporal discount factor	0.96
$\bar{w}$	Average wage rate	0.1
$\bar{\delta}$	Average capital depreciation rate	0.05
r	Real interest rate	0.04
$\mho$	Unemployment rate	0.07
σ_w	Standard Deviation of wage shocks	0.1
σ_δ	Width of depreciation risk band	0.05

3.5 Results

3.5.1 Basis Function

As discussed in the technical background, any completely monotone function would form a valid basis. While there are several standard basis, this paper uses the Gaussian function $\psi(x) = e^{-r^2}$.

3.5.2 Calibration and Methods

As this paper builds on previous work, the calibrated values are those used in White (2015) and presented in Table 3.1.

I solve the model using a variety of grid sizes, with the bounds of the grid being between 0 and 300. At each grid size, both the exogenous and endogenous versions of the model are solved. The full model includes seven states for wages (plus one for unemployment) and eight states for depreciation. The solution time is recorded for each grid size.

To evaluate the accuracy of the policy, I compare the policy functions generated by each solution, relative to a dense grid solved using the benchmark exogenous grid method. I take a fixed set of grid points, and use interpolation to identify the expected policy of each solution method. The results are provided in Table 3.2.

As cam be seem from Table 3.2, the endogenous grid method using RBFs suffers from a significant time penalty when comparing it to standard methods of similar grid size. However, it makes up for this time penalty in terms of the improved accuracy it achieves. For smaller grid counts, this trade-off is potentially one worth considering, but for larger

Table 3.2: Comparison of Solution Methods

Grid	Timing (seconds, relative to EXOG)		Accuracy (# digits, relative to EXOG 300)	
	EXOG	ENDG	EXOG	ENDG
25x25	(6.9, 1x)	(15.2, 0.45x)	2.1	3.1
50x50	(40.1, 1x)	(90.3, 0.36x)	3.7	4.2
100x100	(174.2, 1x)	(617.3, 0.28x)	3.1	5.1
150x150	(402.5, 1x)	(3031, 0.17x)	4.7	6.7
200x200	(685.7, 1x)		5.4	
250x250	(1050, 1x)		7.3	
300x300	(1634, 1x)		8	

grid values, the increased accuracy does not appear to provide sufficient value given the significant time increase.

One potential method of improving results would be to use hardware acceleration. As discussed, the RBF method involves solving matrices, something that can be implemented efficiently on a graphical processor. There are, however, limitations to the sizes of the grid in these situations, given the significantly lower memory available in graphics cards. However, with some tweaking, this may be a method of increasing performance. This is left for future work.

3.6 Conclusions

This paper presents a new technique for solving the endogenous grid problem in multiple dimensions. Using RBFs, it presents a basis function that permits approximation of the model. While the accuracy achieved with this method is significant, it significantly reduces the benefit of using the endogenous grid - namely speed improvements. There are several potential cases where this technique would be useful, however, such as where the number of dimensions is high.

Finally, this paper has provided the technique primarily as a proof of concept. There may be significant speed gains to be achieved by using graphical processors and parallelization techniques. Additionally, further exploration of basis functions could yield significant speed or accuracy improvements.

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Appendix A

Analytical Derivations

A.1 Chapter 1: OLG Model

A.1.1 Solving for Wages with linear utilities

Decompose the wage for generation i as follows

$$w_i\left(z, \bar{w}\right) = b + \Delta_i\left(z, x, \bar{w}\right)$$

where Δ_i is the incremental value over the outside option that an agent of type i with previous wage $\bar{w}$ receives. From equations (1.14) and (1.15) we have

$$\begin{aligned} E_i\left(z, x, \bar{w}\right) - U_i\left(z, x\right) &= \Delta_i\left(z, x, \bar{w}\right) + \\ &\quad + \beta \mathbb{E}_z\left[\left(1 - s - f\left(\theta_z\right)\right)\left(E_{i+1}\left(z', x+1, w_i\right) - U_{i+1}\left(z', x+1\right)\right)\right] \\ &\quad + \beta \mathbb{E}_z\left[f\left(\theta_z\right)\delta_{i+1}^{E-U}\left(z', x+1, x-1, w_i, b\right)\right] \\ \delta_i^{E-U}\left(z, x, y, \bar{w}, b\right) &= \left[E_i\left(z, x, \bar{w}\right) - U_i\left(z, x\right)\right] - \left[E_i\left(z, y, b\right) - U_i\left(z, y\right)\right] \end{aligned} \tag{A.1}$$

Similarly, we will decompose equation (1.7) as follows:

$$J_i\left(z, \bar{w}\right) = J_i\left(z, b\right) + \delta_{i,j}^{J}\left(z, \bar{w}, b\right)$$

Combining equations (1.13a) and (A.1) we get

$$\begin{aligned}\Delta_i(z,x,\bar{w}) &= E_i(z,x,\bar{w}) - U_i(z,x)\\ &\quad -\beta\mathbb{E}_z\left\{\left(1-s\left(z'\right)-f\left(\theta_z\right)\right)\left(E_{i+1}\left(z',x+1,w_i\right)-U_{i+1}\left(z',x+1\right)\right)\right\}\\ &\quad +\beta\mathbb{E}_z\left\{f\left(\theta_z\right)\delta_{i+1}^{E-U}\left(z',x+1,x-1,w_i,b\right)\right\}\end{aligned}$$

Therefore, again using the bargaining solution, we have

$$\begin{aligned}\Delta_{J-k}(z,x,t,w_{-1}) &= \frac{\gamma}{1-\gamma}J_{J-k}(z,x,t,w_{-1}) -\\ &\quad \beta\mathbb{E}_z\left\{\left(1-s\left(z'\right)-f\left(\theta_z\right)\right)\left(\frac{\gamma}{1-\gamma}\right)J_{J-k+1}\left(z',x_e',t+1,w_{J-k}\right)+\right.\\ &\quad \left. f\left(\theta_z\right)\frac{\gamma}{1-\gamma}\delta_{J-k+1}^{J}\left(z',x_e',x_u',t+1,w_{J-k},b\right)\right\}\\ &= \frac{\gamma}{1-\gamma}\left(y_{J-k}(z,t)-w_{J-k}(z,x,t,w_{-1})-h\left(w_{J-k},w_{-1}\right)\right)-\\ &\quad \frac{\gamma}{1-\gamma}\beta\mathbb{E}_z\left[s\left(z'\right)J_{J-k+1}\left(z',x',t+1,w_{J-k}\right)\right]+\\ &\quad \frac{\gamma}{1-\gamma}\beta\mathbb{E}_z\left\{f\left(\theta_z\right)J_{J-k+1}\left(z',x',0,b\right)\right\}\end{aligned}$$

Rearranging terms yields

$$\begin{aligned}w_{J-k}(z,x,t,w_{-1})+\gamma h\left(w_{J-k},w_{-1}\right) &= (1-\gamma)b+\gamma y_{J-k}(z,t)+\\ &\quad \gamma\beta\mathbb{E}_z\left\{f\left(\theta_z\right)J_{J-k+1}\left(z',x',0,b\right)\right\}\end{aligned}$$

A.1.2 Comparative Statics: Solving for elasticity

$$\begin{aligned}\frac{c}{\beta q\left(\theta_z\right)} &= \mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)\\ \frac{c}{\beta} &= \frac{f\left(\theta_z\right)}{\theta_z}\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)\end{aligned}$$

$$0=\frac{1}{\theta}\frac{d\theta}{d(y-b)}\left(\frac{df}{d\theta}\frac{\theta}{f}-1\right)\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)+\frac{\partial\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)}{\partial(y-b)}+\frac{\partial\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)}{\partial\theta}\frac{d\theta}{d(y-b)}$$

$$\frac{1}{\theta}\frac{d\theta}{d(y-b)}\left[(1-\eta)\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)-\theta\frac{\partial\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)}{\partial\theta}\right]=\frac{\partial\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)}{\partial(y-b)}$$

$$\frac{y-b}{\theta}\frac{d\theta}{d(y-b)}=\frac{(y-b)\left(\frac{\partial\mathbb{E}_z\sum_j \pi_j J_j(z')}{\partial(y-b)}\right)}{(1-\eta)\mathbb{E}_z\sum_j \pi_j J_j\left(z'\right)-\theta\frac{\partial\mathbb{E}_z\sum_j \pi_j J_j(z')}{\partial\theta}}$$

A.1.3 Wages for lifecycle model

We have

$$\begin{aligned} U_{J-k-1}(z) &= b+\beta\mathbb{E}_z\left\{U_{J-k}\left(z'\right)+f\left(\theta_{z'}\right)\left[E_{J-k}\left(z'\right)-U_{J-k}\left(z'\right)\right]\right\} \\ E_{J-k-1}(z) &= b+\Delta_{J-(k+1)}(z)+\beta\mathbb{E}_z\left\{E_{J-k}\left(z'\right)-s\left[E_{J-k}\left(z'\right)-U_{j-k}\left(z'\right)\right]\right\} \end{aligned}$$

We can rewrite this as

$$\begin{aligned} E_{J-k-1}(z)-U_{J-k-1}(z) &= \Delta_{J-(k+1)}(z)+\beta\mathbb{E}_z\left[\left(E_{J-k}\left(z'\right)-U_{J-k}\left(z'\right)\right)\left(1-s-f\left(\theta_{z'}\right)\right)\right] \\ &= \Delta_{J-(k+1)}(z)+\beta\mathbb{E}_z\left[\left(\Delta_{J-k}\left(z'\right)+\sum_{i=1}^{k-1}\beta^{k-i}\mathbb{E}_{z'}\right.\right. \\ &\quad \left.\left.\left[\Delta_{J-i}\left(z^{(k+1-i)}\right)\prod_{m=1}^{k-i}\left(1-s\left(z^{(m+1)}\right)-f\left(\theta_{z^{(m+1)}}\right)\right)\right]\right) \\ &\quad \left(1-s-f\left(\theta_{z'}\right)\right)\right] \\ &= \Delta_{J-(k+1)}(z)+\sum_{i=1}^{k}\beta^{k+1-i}\mathbb{E}_z \\ &\quad \left[\Delta_{J-i}\left(z^{(k+1-i)}\right)\prod_{m=1}^{k+1-i}\left(1-s\left(z^{(m)}\right)-f\left(\theta_{z^{(m)}}\right)\right)\right] \end{aligned}$$

From the firm problem, we have

$$J_{J-k}(z) = y(z)-w_{J-k}(z)+\sum_{i=1}^{k-1}\beta^i\mathbb{E}_z\left[\left(y\left(z^{(i)}\right)-w_{J-k+i}\left(z^{(i)}\right)\right)\prod_{m=1}^{i}\left(1-s\left(z^{(m)}\right)\right)\right]$$

Using the wage equation from Appendix A.1.1 and Nash Bargaining, we can then show that

$$\begin{aligned} w_{J-k}(z) = & \quad (1-\gamma)\,b+\gamma y(z)+ \\ & \gamma(1-\gamma)\textstyle\sum_{i=1}^{k-1}\beta^i\mathbb{E}_z\left[f\left(\theta_{z'}\right)\left(y\left(z^{(i)}\right)-b\right)\frac{\prod_{j=1}^{i}1-s\left(z^{(j)}\right)-\gamma f\left(\theta_{z^{(j)}}\right)}{1-s\left(z'\right)-\gamma f\left(\theta_{z'}\right)}\right] \end{aligned}$$

A.2 Chapter 1: Model 2

A.2.1 Comparative Statics: Solving for elasticity

$$\frac{c}{\beta q(\theta_z)} = \int_{z'} \left\{ \int_{\delta_i(z')} g(i|z,z') J_i(z') di \right\} d(z'|z)$$
$$\frac{c}{\beta} = \frac{f(\theta_z)}{\theta_z} \int_{z'} \left\{ \int_{\delta_i(z)} g(i|z,z') J_i(z') di \right\} d(z'|z)$$

Taking the derivative,

$$0 = \frac{1}{\theta} \frac{d\theta}{d(y-b)} \left(\frac{df}{d\theta} \frac{\theta}{f} - 1 \right) \mathbb{E}\left\{ \mathbb{E}_{z'} J_i(z') \right\} + \frac{\partial \mathbb{E}\left\{ \mathbb{E}_{z'} J_i(z') \right\}}{\partial (y-b)} + \frac{\partial \mathbb{E}\left\{ \mathbb{E}_{z'} J_i(z') \right\}}{\partial \theta} \frac{d\theta}{d(y-b)}$$

$$\frac{1}{\theta} \frac{d\theta}{d(y-b)} \left[(1-\eta) \mathbb{E}\left\{ \mathbb{E}_{z,z'} J_i(z') \right\} - \theta \frac{\partial \mathbb{E}\left\{ \mathbb{E}_{z'} J_i(z') \right\}}{\partial \theta} \right] = \frac{\partial \mathbb{E}\left\{ \mathbb{E}_{z,z'} J_i(z') \right\}}{\partial (y-b)}$$

$$\frac{y-b}{\theta} \frac{d\theta}{d(y-b)} = \frac{(y-b)\left(\frac{\partial \mathbb{E}\left\{ \mathbb{E}_{z,z'} J_i(z') \right\}}{\partial (y-b)} \right)}{(1-\eta) \mathbb{E}\left\{ \mathbb{E}_{z,z'} J_i(z') \right\} - \theta \frac{\partial \mathbb{E}\left\{ \mathbb{E}_{z,z'} J_i(z') \right\}}{\partial \theta}}$$

A.3 Chapter 2: Equivalent Economy

Our economy with mitigated shocks is equivalent to an economy that features state-contingent claims and an enforcement constraint on how much income above the lowest possible income state can be hidden. First, calculate the expected value of bond holdings:

$$\bar{b_t} = \sum_{s_{t+1}} b(s_{t+1}) g(s_t, s_{t+1})$$

Then rewrite individual bond holdings as:

$$b(s_{t+1}) = \bar{b_t} + x(s_{t+1})$$

Where:

$$\sum_{s_{t+1}} x(s_{t+1})g(s_t, s_{t+1}) = 0$$

Then, the evolution of assets is able to be rewritten as:

$$a(s_{t+1}) = y_{t+1} + \sum_{l=1}^{N} \left[k_{l,t} + z_{l,t+1}k_{l,t}^{\nu}\right] + \bar{b_t} + x(s_{t+1})$$

Next, rewrite the realized shock value j in relation to the lowest shock state 1:

$$a(s_j) = a(s_1) + (1 - \phi^i) \left[w(s_j) - w(s_1) + \sum_{l=1}^{N}(z_l(s_j) - z_l(s_1))k_l^{\nu}\right]$$

Now, substitute in the law of motion to get:

$$x(s_j) - x(s_1) = -\phi^i \left[w(s_j) - w(s_1) + \sum_{l=1}^{N}(z_l(s_j) - z_l(s_1))k_l^{\nu}\right]$$

The above holds $\forall j \in \{2, ..., J\}$, where J is the number of possible shock states. Recall that $\sum_j x(s^j)g(s_t, s^J) = 0$. Then:

$$x(s_j) = -\phi^i W^j(s_t) - \phi^i \sum_{l=1}^{N} Z_l^j(s_t)k_l^{\nu}$$

Where:

$$W^j(s_t) = w^j - \sum_i g(s_t, s_i)w^i$$

And:

$$Z_l^j(s_t) = z_l^j - \sum_i g(s_t, s_i)z_l^i$$

Define:

$$\tilde{w}^j(s_t) = w^j - \phi W^j(s_t)$$

$$\tilde{z}_l^j(s_t) = z_l^j - \phi Z_l^j(s_t)$$

Then, the formulation used in the model is obtained.

$$a(s^j) = \tilde{w}^j(s_t) + \sum_{l=1}^{N} \left[k_{l,t} + \tilde{z}_l^j(s_t) k_{l,t}^{\nu} \right] + \bar{b_t}$$

A.4 Chapter 2: Characterization Algebra

To characterize the solution to this problem, first take the first order conditions with respect to each choice variable.

$$\begin{aligned}
c: &\qquad \lambda = u'(c) \\
k_1': &\qquad \beta E\left[\frac{\partial V'}{\partial k_1'}\right] - \lambda + E\left[\mu(s')\frac{\partial a(s')}{\partial k_1'}\right] = 0 \\
k_2': &\qquad \beta E\left[\frac{\partial V'}{\partial k_2'}\right] - \lambda + E\left[\mu(s')\frac{\partial a(s')}{\partial k_2'}\right] = 0 \\
b': &\qquad \beta E\left[\frac{\partial V'}{\partial b'}\right] - \lambda + E\left[\mu(s')\frac{\partial a(s')}{\partial b'}\right] = 0
\end{aligned}$$

After calculating the first order conditions, we need the envelope conditions for each choice variable. Note that, for all choice variables $\{k_1', k_2', b'\} \in x$, we must use the chain rule: $\frac{\partial V}{\partial x} = \frac{\partial V}{\partial a}\frac{\partial a}{\partial x}$. Taking these envelope conditions yields:

$$\frac{\partial V}{\partial a} = \lambda$$

$$\frac{\partial a(s)}{\partial k_1'} = (\alpha \tilde{z}_1 k_1^{\alpha-1} + 1)$$

$$\frac{\partial a(s)}{\partial k_2'} = (\alpha \tilde{z}_1 k_1^{\alpha-1} + 1)$$

$$\frac{\partial a(s)}{\partial b'} = (1 + R(A_1^-, A_2^-))$$

$$\frac{\partial V}{\partial k_1} = \lambda(\alpha \tilde{z}_1 k_1^{\alpha-1} + 1)$$

$$\frac{\partial V}{\partial k_2} = \lambda(\alpha \tilde{z}_1 k_1^{\alpha-1} + 1)$$

$$\frac{\partial V}{\partial b} = \lambda(1 + R(A_1^-, A_2^-)$$

Now that we have the components, it simply remains to plug the envelope conditions into the first order conditions to generate the Euler equations:

$$\beta E\left[u'(c')(\alpha \tilde{z'}_1 k_1'^{\alpha-1} + 1)\right] + E\left[\mu(s')\alpha(\tilde{z'}_1 k_1'^{\alpha-1} + 1)\right] = u'(c)$$

The Euler equation for capital in country 2 is:

$$\beta E\left[u'(c')(\alpha \tilde{z'}_2 k_2'^{\alpha-1} + 1)\right] + E\left[\mu(s')\alpha(\tilde{z'}_2 k_2'^{\alpha-1} + 1)\right] = u'(c)$$

The Euler equation for bonds is:

$$\beta E\left[u'(c')(1 + R(A_1, A_2))\right] + E\left[\mu(s')(1 + R(A_1, A_2)\right] = u'(c)$$